FINANCIAL RISK FOR THE BENEFICIARY IN NOTIONAL DEFINED CONTRIBUTION ACCOUNTS*.

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ABSTRACT
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This paper aims to quantify the *aggregate financial* risk to which the beneficiary would be exposed if it were decided to introduce a retirement pension system based on notional account philosophy in Spain. For this we use scenario generation techniques to make projections of the factors determining the real expected internal rate of return (IRR) for the beneficiary according to sixteen retirement formulae based on the most widely accepted rates or indices. These projections are based on Herce and Alonso's macroeconomic scenario 2000-2050 (2000a) and include information about the past performance of the indices and the time period for which the forecast is wanted. The results of the IRR calculation - average value, standard deviation, value-at-risk (VaR) - are analyzed both in objective terms and for different degrees of participants' risk aversion. (JEL: H55, J26)

KEYWORDS: Spain, Retirement, Pay-as-you-go, Internal Rate of Return (IRR), Beneficiary risk.

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1.-INTRODUCTION

The race to reform pension systems in many countries over the last few years has been such that, as Valdés-Prieto (2002) points out, the problems of pension reform have begun to dominate economic policies. Information relating to the reforms carried out, by area or by country, can be found in papers by Börsch-Supan, Palacios and Tumbarello (1999), Devesa and Vidal (2001), Fox and Palmer (2000 and 2001), Lindeman, Rutkowsky and Sluchynsky (2001), Müller (2001a and 2001b), Palacios and Pallarés (2000), Schwarz and Demirguc-Kunt (1999), and Social Security Administration (2002).

The main reforms proposed and applied can be summarized as parametric reforms of the pay-as-you-go (PAYG) system, changes to other (mainly capitalization) systems, and systems combining capitalization and PAYG, as proposed chiefly by the World Bank. Reform trends championed by the main international organizations can be found in papers by Gillion (2000), Holzmann (2000) and Queisser (2000).

One of the most important recent innovations in pension reform has been the introduction of so-called "notional defined contribution accounts" in some countries, namely Brazil[3] (1999), Italy (1995), Latvia (1996), Poland (1999) and Sweden (1999). This type of retirement formulation is considered suitable for those countries where, due to special demographic or political conditions, it is difficult to introduce an at least partial accumulation of funds. The system establishes an analogy between the PAYG and capitalization systems by incorporating actuarial and financial instruments used in the capitalization system into the PAYG system. According to Valdés-Prieto (2000), this strengthens the long-term financial solvency of the PAYG system but increases the uncertainty surrounding the pension to be received by the beneficiary[4].

The European Union[5], the World Bank and the OECD along with various researchers such as Bonin, Gil and Patxot (2001), Devesa, Lejárraga and Vidal (2000 and 2002), Gil and Patxot (2002), Herce (1997), Herce and Alonso (2000a and 2000b), Jimeno and Licandro (1999), Jimeno (2002), Mateo (1997), Meneu (1998), Montero (2000) and Piñera and Weinstein (1996) have all strongly recommended an in-depth revision of the Spanish public pension system. All are agreed that, at least in the long term, the financial viability of the system is seriously at risk.

One valid possibility could be the introduction of notional accounts, transforming the system from defined benefit to defined contribution as suggested by Jimeno (2002). Vidal, Devesa and Lejárraga (2002) have studied the effect that the introduction of various notional retirement formulae similar to those actually applied in some countries would have had in Spain. They concluded that it would have noticeably decreased the amount of the pensions currently being paid, which are based on a traditional defined benefit formula. The real internal rate of return expected from the contributions would also have decreased from around 6% to less than 2.5% under any of the formulae applied. These values are more in line with the 3% real average GDP growth in Spain over the last thirty years, which should undoubtedly be the benchmark to aim at for the system to be financially viable in the Samuelson sense.

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[3] This is not exactly a notional accounts system.

[4] This happens because it is a defined contribution system.

[5] The latest from the European Commission is for December 2002. A survey of the most recent papers by various international organizations can be found in the paper by Rother, Catenaro and Schwab (2003).

This paper will concern itself with estimating the *aggregate financial* risk[6] the beneficiary would be exposed to if it were decided to introduce a retirement pension system based on notional accounts in Spain. After this introduction, in the next section we will define the concept of "notional defined contribution accounts". In the third section we set out the projection model, which includes information obtained from Herce and Alonso's macroeconomic scenario 2000-2050 (2000a), information about the past performance of the indices, and information relating to the time period the projection is to be made for. In the fourth section we use scenario generation techniques to present projections of the expected internal rate of return (IRR) for the beneficiary using sixteen notional retirement formulae linked to the retail price index (RPI), the real gross domestic product (GDP), the average earnings index (AEI), and the total Social Security contributions index (TSSCI). The results of the IRR calculations - average value, standard deviation, value-at-risk (VaR) - are analyzed both in objective terms and for different degrees of participant's risk aversion via a function that relates the average, the standard deviation and a risk aversion coefficient. The paper ends with the main conclusions reached, full references, and finally an appendix which concisely sets out the actuarial evaluation of notional defined contribution retirement accounts.

II.-THE NOTIONAL ACCOUNTS MODEL

According to Vidal, Devesa and Lejárraga (2002), a notional account is a virtual account in which the contributor's individual contributions are collected along with the fictitious returns these contributions generate throughout the contributor's working life. Returns are calculated according to a notional rate, which could be the growth rate of the GDP, of average earnings, the wage bill, the income from total Social Security contributions, etc. When people retire they receive a pension based on the accumulated notional fund, the specific mortality rate for the cohort retiring that year, and the notional rate used. In Spain, Mateo (1997) was the first to approach the concept of virtual accounts in the proposal he put forward for the general redesign of the pension system.

At first sight notional defined contribution plans appear to be just an alternative way of calculating the amount of retirement pensions. The account is called notional because it exists only on paper. Money is not deposited in any real account. Nevertheless, the amount of the pension is based on the fund accumulated in the notional account. Contributions made to notional accounts are capitalized at a notional rate of return. This hypothetical return is normally linked to some external index set by law. Whatever the index used, the contributions are capitalized at a hypothetical rate of return, although this is expressed only on paper.

In all the countries that use notional accounts, the hypothetical account is converted into a life annuity on retirement. There would be no difficulty, however, in converting it into a different type of benefit. The conversion is normally done by dividing or multiplying the fund by a set conversion factor, often called the g-value, which depends on life expectancy at the retirement age chosen and the interest rate. Indirectly this has the effect of reducing the degree to which returns vary between generations. The basis for calculating the conversion factor must be set by law. It must also be decided which mortality table and interest rate should be used for the calculation, and whether to separate the conversion factors between men and women, as is done in real capitalization, or whether some common conversion factor should be used to average out life expectancy for men and women together, which is what usually happens in traditional PAYG systems.

[6] It is not the political risk of notional accounts that is measured, though this clearly exists, but rather the aggregate financial risk since what is analyzed is the evolution of average salaries.

Table 1: Main features of the countries analyzed with notional defined contribution accounts.

Features/Countries	BRAZIL (1999)	ITALY (1995-1997)	LATVIA (1996)	POLAND (1999)	SWEDEN (1998-1999)
Current pension system structure	Two-pillar system: 1.-Mandatory defined contribution (31%) pay-as-you-go system and notional retirement formula for private sector employees. Traditional pay-as-you-go with many privileges for civil servants. 2.-Complementary capitalization system organized through companies (moderately developed).	Two-pillar system: 1.-Mandatory pay-as-you-go system with defined contributions (33%) and notional retirement formula. 2.-Incipient company-based complementary capitalization system.	Three-pillar system: 1.-Financed by pay-as-you-go, organized through notional accounts (20%). 2.-Mandatory, comprising individual capitalization accounts, begun in 2001 and will grow in importance. 3.-Voluntary, based on group plans, at present barely developed.	Three-pillar system: 1.- Mandatory pay-as-you-go with notional philosophy (12.22%). 2.- Mandatory capitalization, less important at the start (7.3%). 3.- Voluntary capitalization.	Three-pillar system: 1.-Financed by pay-as-you-go, organized through notional defined contribution accounts (16.5% of contributions) 2.-Mandatory, comprising individual capitalization accounts (2.5%). 3.-Complementary, though very widespread, based on employer schemes.
Notional rate of return on contributions	Endogenous rate according to years contributed and retirement age.	Five-year average based on nominal GDP growth rate.	Growth rate of the total contribution base.	75% of salary growth.	Real growth rate of per capita salaries.
Main features of the retirement pension formula in pillar 1	Average of the 80% highest salaries, adjusted for inflation, divided by common life expectancy, multiplied by an implicit financial factor which depends on retirement age, number of contributions and contribution rate. Annual review of mortality tables.	Standard formula with conversion factor which includes survivor contingency. Retirement age from 57. Real interest rate of 1.5%. Ten-year review of mortality tables.	Standard formula, similar to Equation 14, with common mortality tables, guaranteed minimum pension at 62, and contributions credited for certain periods.	Standard formula, similar to Equation 14, with common mortality tables, guaranteed minimum pension at 60 for women and 65 for men, and contributions credited for certain periods.	Standard formula with common mortality tables, guaranteed minimum pension at 61 and contributions credited for periods of unemployment, sickness and temporary incapacity. Real rate of interest 1.6%.
Notional rate for pensions	Index of growth of minimum wage.	Retail price index.	Combination of price index and salaries.	Retail price index plus 20% of real salary growth.	Retail price index plus/minus an adjustment for the difference between the real growth in salaries and that predicted.
Transitional measures	Yes, gradual application due to deficiencies in contribution records.	Yes, three retirement scenarios are superimposed: Amato Scheme, pro rata scheme and Dini Scheme (entry into labor market from 1-01-96). Will be fully functioning in 2035.	Yes, visible mainly in the way the initial notional capital is determined. Problem with unreliable records.	Yes, the new formula for calculating pensions will not be fully in force until 2014 for men and to a certain extent from 2009 for women.	Yes, the new formula will only be applied 100% to those born from 1954. Will be fully in force before 2020.

Source: Vidal, Devesa and Lejárraga (2002)

The conversion factors are not based on the same elements used by insurance companies, since no annuity is actually bought from an insurer. The factor used in these systems is a mechanism for converting the accumulated fund into a lifetime annuity. Nevertheless, this calculation has a real impact since it determines the pension that will actually be paid to the beneficiaries when they become pensioners at retirement age.

Although the theory behind the notional account system seems clear, there is no single formula to be applied[7]. Each country has "designed" one mathematical expression to calculate the notional amount accumulated for each individual and another one to determine their pension.

Table 1 gives a brief analysis of the most relevant aspects applied in various countries where retirement formulae based on the notional model have been introduced. It compares how the pension systems are organized, the notional rates of return applied to capitalize the contributions, the basic features of the retirement pension formulae, how pensions in payment are adjusted, and the measures that each country has established for making the transition from a defined benefit to a defined contribution system.

III.-FORECAST AND SIMULATION MODEL

Given the basic aim of this paper - to quantify the risk any beneficiary that enters the labor market will be exposed to if it were decided to introduce a retirement pension system based on notional accounts in Spain - very long-term projections of macroeconomic variables will have to be made, since the projection period under consideration covers from the time a generation of 25-year-olds enters the labor market until the last pensioner dies, approximately 101 years later. As Herce and Alonso (2000a) point out, establishing a macroeconomic scenario for 2050 is even more difficult than making demographic projections. Judging by the way the Spanish economy has changed over the last fifty years, any forecast for the next fifty is bound to be highly debatable. For Alvarez, Ballabriga and Jareño (1997), the complexity of economic reality makes economic forecasting an inherently difficult exercise, and this difficulty is reflected in the high level of uncertainty that normally accompanies it. Bearing this in mind, the logical attitude should be to try to adequately define the uncertainty rather than ignore it and give a false impression of rigor and accuracy.

Ballabriga et al (1998) put forward a macroeconometric model for the Spanish economy based on Bayesian Vector Autoregressive methodology (BVAR). This model reveals the existence of both short and long-term relationships between the macroeconomic variables. Using this model to forecast is done by considering three annual periods. In the paper it is found that the number of parameters to be estimated grows exponentially as the number of variables and the time lag considered increase.

The risk analysis carried out in this paper includes considering a time scale of 101 years, and therefore the BVAR model will not help us reach our goal. This same justification would be valid when estimating any autoregressive model with mean reversion.

The aim is not so much to estimate the future value of the parameters, but rather to evaluate the consequences for the beneficiary of a notional accounts system in an uncertain environment where the variables influencing the system follow different behaviors. For this reason no econometric model is estimated. Instead the work is based on a more intuitive model generated by the discrete form of an additive Brownian behavior of the parameters, without affecting its including information about the past performance of the parameters.

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[7] See Appendix for the actuarial evaluation.

According to Devolder (1993), the model used to obtain different tracks of behavior for the relevant indices (macroeconomic variables) is the following[8]:

$$I_t^s = \mu_{I,t} \pm \lambda^s \sigma_I \sqrt{t} \qquad [1.]$$

$\forall I = RPI, GDP, AEI, TSSCI$

where:

I_t^s : Value of index "I", in period "t" and under scenario "s".

$\mu_{I,t}$: Average value of index "I" in period "t".

λ^s : Parameter generating scenario "s".

σ_I : Typical deviation of index "I".

RPI : Index of annual variation in retail prices.

GDP : Index of annual variation in gross domestic product.

AEI : Index of annual variation in average earnings.

TSSCI : Index of annual variation in total Social Security contributions.

This formulation is suitable for making forecasts, since:

1) Information based on Herce and Alonso's macroeconomic scenario 2000-2050 (2000a) is included, reflected in the model through parameter $\mu_{I,t}$. In other words it is interpreted as the average value of the index in question for each of the periods analyzed. This value includes the behavioral trend of the parameter.

2) Information about the past performance of the indices is incorporated through parameter σ_I, which gives information on the typical deviation of the index in the historical series analyzed.

3) λ^s is a parameter generating the different scenarios.

4) With regard to long-term relationships, we suppose a perfect correlation between all the economic factors, and for this we use the same value of parameter λ^s for all the different indices. For instance, if the GDP is high for one scenario, then so is the RPI.

5) Finally, information relating to the period of time the projection is made for is included through term $\sqrt{t}$ [9].

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[8] Work has also been done with a formula that responds to a geometric Brownian motion:

$$I_t^S = \mu_{I,t} \exp\left[-\frac{\sigma_I^2}{2} t \pm \lambda^S \sigma_I \sqrt{t} \right]$$

. However, since these are very long-term forecasts, the effects of the positive scenarios are overrated while the effects of the negative scenarios are undervalued. With the geometric model, if the starting point is a positive value, then negative values can never be arrived at. This is unreal since negative growth rates of the GNP or nominal salaries could exist in very unfavorable scenarios.

[9] This assumption is in relation to assumption 2, because we are working with linear variance and independent increments.

The model enables projections, known as scenarios, to be made of the behavior of the parameters. Each of these scenarios, $\forall s = 1,2,,....S$, has an associated probability of occurrence equal to p^s, $p^s>0$ y $\sum_{s=1}^{S} p^s = 1$. There are two ways of working with scenarios:

1) those obtained through a distribution where a series of random numbers are generated; or

2) those that follow a distribution whose different parameters are known.

In the first case the optimal number of random number scenarios according to Smith and Southall (2001) and Mulvey and Ziemba (1998) is around 10,000 simulations. In the second case the number is lower, and so the simulation is much easier. Scenarios generated as in the second case are used in this paper.

The most relevant data and assumptions when carrying out a simulation are as follows:

1) Number of scenarios S=20

2) Values of the generating parameter: identical increases and decreases relative to the average value have been assumed.

Table 2: Values of the generating parameter.

s	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
λ^s	.01	.02	.03	.04	.05	.06	.07	.08	.09	.1	-.01	-.02	-.03	-.04	-.05	-.06	-.07	-.08	-.09	-.1

3) Probability of each scenario: a distribution that assigns a greater probability of occurrence to those values closer to the average has been assumed[10].

Table 3: Probability of the scenarios.

s	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
p^s	.085	.077	.069	.062	.054	.046	.038	.031	.023	.015	.085	.077	.069	.062	.054	.046	.038	.031	.023	.015

4) Periods: t = 0,1,... 101. Age of contributor[11] from 25 (entry into the labor market) to 126 (final age appearing in GRMF-95 mortality tables).

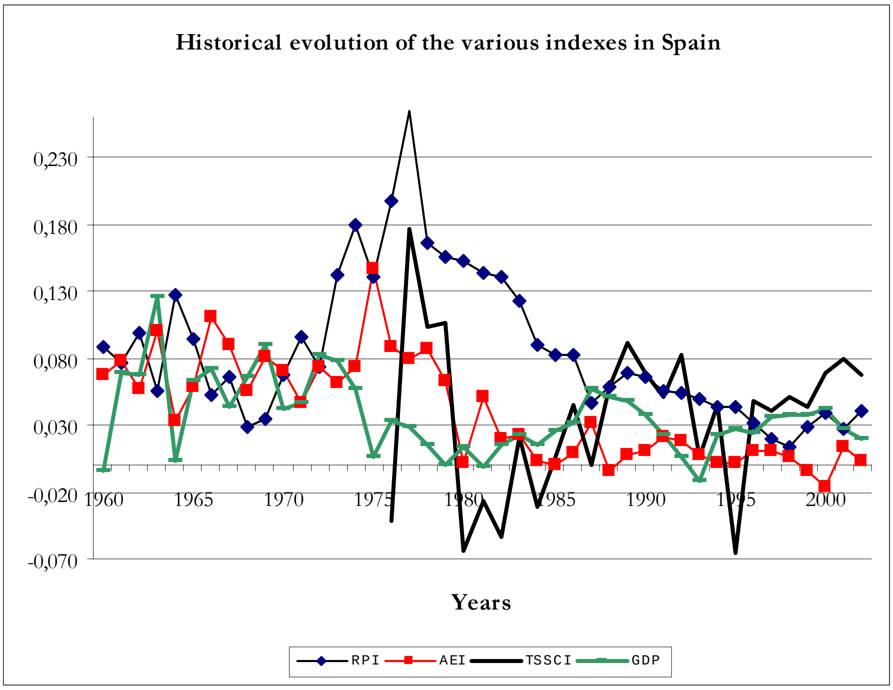
5) Years of evaluation: 2003, 2004,...2104

6) Average value and past deviations of the parameters, in real terms:

[10] Tests have also been carried out with two more distributions in order to test the sensitivity of the results to the distribution used in generating the scenarios: 1.-Uniform distribution. 2.-Negative-biased distribution, assigning greater probability of occurrence to the scenarios that assume a decrease with respect to the average. The numerical results obtained are very similar to the uniform distribution, with the same classification order maintained. With the negative-biased distribution, despite the fact that the results are also very similar, the classification order is different. However, the same models still appear in the first five places. See Section IV.

[11] A study of past employment figures representative of the different Social Security contribution groups in Spain indicate contribution periods that barely cover 35 years. According to figures published for Spain, the average age for entering the jobs market is around 25.

Table 4: Average values and past deviations of the various indices in real terms.												
Past deviations[12]	**Year**	**2003**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**	**2035**	**2040**	**2045**	**>2050**
	t	**0**	**3**	**7**	**12**	**17**	**22**	**27**	**32**	**37**	**42**	**>47**
σ_{GDP} : 0.0219	$\mu_{GDP,t}$	0.037	0.030	0.030	0.030	0.030	0.028	0.019	0.016	0.013	0.017	0.023
σ_{RPI} : 0.0434	$\mu_{RPI,t}$	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
σ_{AEI} : 0.0337	$\mu_{AEI,t}$	0.008	0.014	0.018	0.021	0.023	0.024	0.024	0.024	0.024	0.024	0.024
σ_{TCSSI} : 0.0452	$\mu_{TCSSI,t}$	0.035	0.027	0.026	0.025	0.024	0.022	0.013	0.010	0.007	0.011	0.017
Source: Authors, based on Herce and Alonso (2000a)												

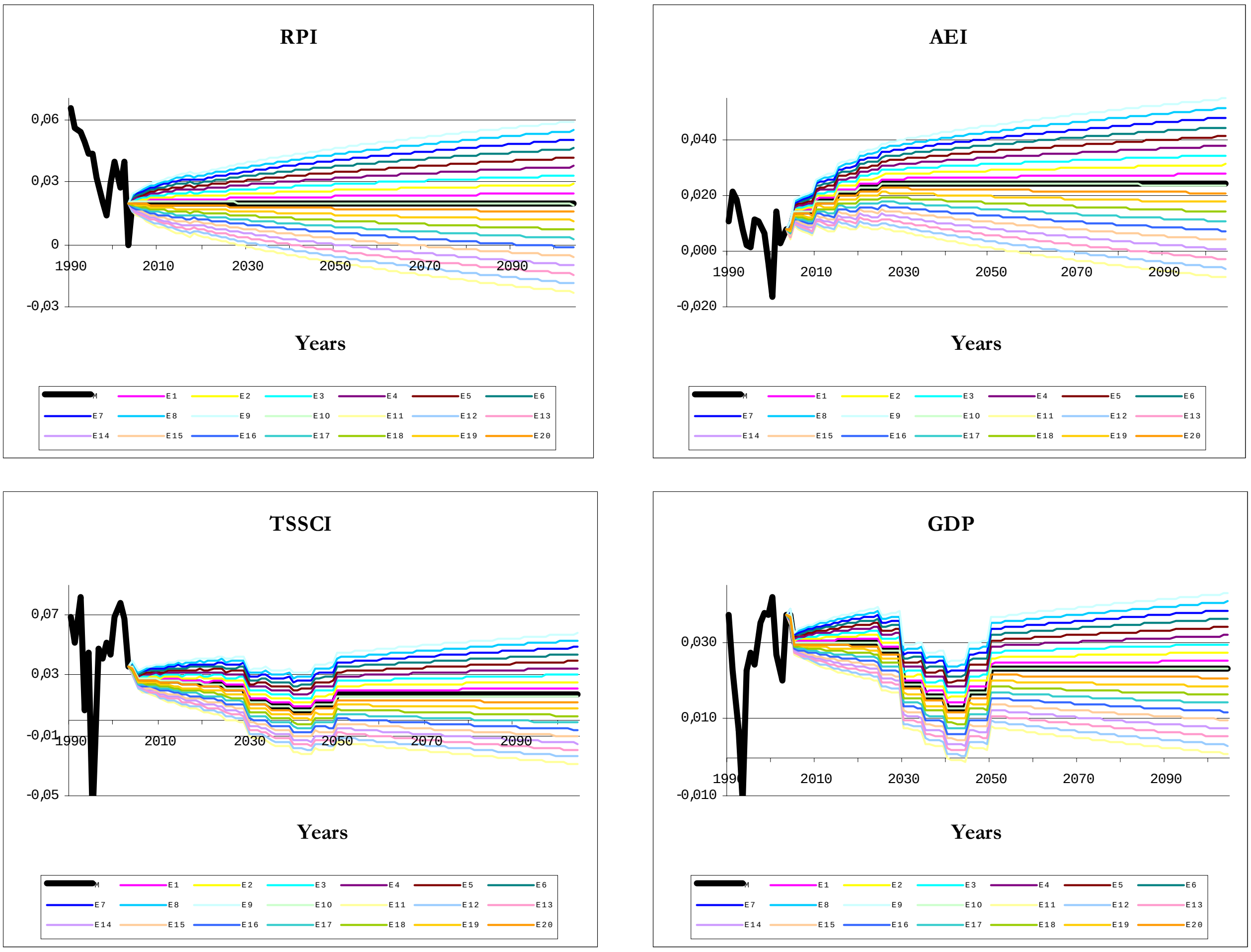


Graph 1: Past performance of the various indices in Spain.

Source. RPI: Instituto Nacional de Estadística (National Institute of Statistics [NIS]). AEI: 1964-1976 (Earnings per hour worked), 1976-1981 (Average monthly salary per working person), 1981-2002 (Average earnings per worker per month) from the Bank of Spain's Statistics Bulletin. TSSCI: Social Security contributions by employed workers (RGSS in Spanish), General Social Security and NIS Treasury Reports. GDP: NIS Statistics Yearbook and the Bank of Spain's Statistics Bulletin.

Graph 2 shows the results of the projections for each of the twenty possible scenarios, for each of the macroeconomic variables in Table 4, and the last twelve annual values. It is interesting to see how the form of the graphs clearly illustrates the increasing uncertainty over time.

[12] See Graph 1.



Graph 2: Evolution and projection of the RPI, AEI, TSSCI, and GDP

Details of how the macroeconomic scenario has been constructed can be found in the paper by Herce and Alonso (2000a). What most attracts the attention is the drastic change in the type of growth of the Spanish economy which, according to the authors, is caused by the great "manpower shortage" the Spanish economy will suffer from 2025. After 2025 it is the growth in productivity that becomes the key indicator of the economy's progress. The decrease in working population will become more and more obvious after 2020, given the stabilization of the rate of employment. Employment will enter a phase of negative growth. The economy will begin to shed jobs. The GDP will grow at a lower rate than productivity, and productivity will take over as the best indicator of the economy's progress.

Real salaries, which are the key for determining contributions as a whole, and employment will be growing at a lower rate than productivity, though following a similar pattern. The GDP deflator, which is assumed to be the same as the rate of inflation, maintains an annual growth rate of 2% throughout the period.

IV.-ANALYSIS OF BENEFICIARY RISK

The beneficiary is subject to risk in that he does not know for certain what the IRR on his contributions will be. The *aggregate financial* risk for the beneficiary is defined as the possibility that the rate of return on the contributions paid may not coincide with the expected rate. To quantify this risk it will be necessary to calculate how the return on the contributions deviates from its expected value. This can be measured either by the typical variance or deviation of the IRR random variable associated with the contributor-beneficiary's contributions, or by the VaR.

The behavior of the indices used as notional adjustment rates (see Table 5) for contributions and pensions is random, and this randomness is taken into account by the scenarios. This means that the IRR obtained from the equivalence between the amounts "tracked" in the notional account and the pensions also has an uncertain behavior pattern associated with each of the scenarios.

To measure beneficiary risk a scenario generation model is used enabling IRR behavior tracks to be projected in the future. This model quantifies the effect of the deviations brought about by the behavior of the real IRR when there are deviations in the parameters that affect its calculation.

Because the analysis of beneficiary risk is closely related to the concept of real IRR, the IRR must be defined precisely. According to Devesa, Lejárraga and Vidal (2002), the apparent a priori expectation of real IRR for a contributor entering the labor market at age x_e in a pure PAYG system with retirement benefits, assuming the system's rules remain constant, is defined as the parameter of value i of the law of compound capitalization which actuarially matches the flow of contributions with the flow of benefits.

The real IRR, for each scenario "s", is determined from the following equation:

$$\sum_{x=x_e}^{x_r-1} RAC_x^s \,(1+IRR^s)^{-(x-x_e)} = \sum_{x=x_r}^{\omega} RAP_x^s \,(1+IRR^s)^{-(x-x_e)} \qquad [2.]$$

RAC^s_x: Real actuarial contribution paid at age "x" under scenario "s"
RAP^s_x: Real actuarial pension received at age "x" under scenario "s".
ω: Age limit on the mortality table.
x_e : Age of entry into the labor market.
x_r: Retirement age.
IRR^s: Internal rate of return under scenario "s".

The value of the real actuarial contribution for a person aged x:

$$RAC_x^s = Cr_x \; W_x^s \; {}_{x-x_e}p_{x_e} \qquad [3.]$$

Cr_x: Contribution rate at age "x". This is assumed to be equal to 15% throughout the period[13].
W^s_x: Salary base at age "x" under scenario "s".
${}_{x-x_e}p_{x_e}$: Probability[14] that an individual of age "x_e" will reach age "x".

The real actuarial pension at age "x" is the real value of the pension affected by the probability of survival from the moment of entry into the labor market:

$$RAP_x^s = P_{x_r}^s \; {}_{x-x_e}p_{x_e} \prod_{t=x_r}^{x} (1+\alpha_t^s) \qquad [4.]$$

[13] This is obtained as an approximation, given that in Spain there is no legally established allocation for retirement contingency. It has been considered that, according to data from the Social Security budget, out of the total contributions for common contingencies applicable in the general employed-worker system, a 15% contribution rate will be assigned to the retirement contingency. As examples we should point out that Jimeno and Licandro (1999) use a 15% contribution rate, Devesa, Lejárraga and Vidal (2002) 14.79%, and Durán (1995) 24.8%. Durán, however, takes into account invalidity and widowhood contingencies as well as retirement.
[14] The mortality table used is GR-95

P^{s}_{xr}: Initial pension (at retirement age x_r), obtained according to the notional capital accumulated under scenario "s".
α_t^s : Index used to increase pensions under scenario "s".

The initial pension at retirement age under scenario "s" is found by[15]:

$$\mathbf{P}_{x_r}^{s} = g\,K^s = g \sum_{t=x}^{x_r-1} Cr_t \cdot W_t^s \prod_{i=t}^{x_r-1}(1+r_i^s) \qquad [5.]$$

where:
K^s: Notional fund accumulated under scenario "s".
r_i^S : Index used to capitalized contributions under scenario "s" at time "i".
g: g-value, the pre-determined conversion factor, which is equal to the inverse of the actual value of a life annuity due of 1 per year, while "x_r" survives, increasing at the accumulative annual rate of β, with I being the technical interest rate.

$$g = \frac{1}{\sum_{t=x_r}^{w} \frac{(1+\beta)^{t-x_r}}{[(1+I)]^{t-x_r}} \; {}_{t\text{-}x_r}p_{x_r}} = \frac{1}{\ddot{a}_{x_r}^{\beta}} \qquad [6.]$$

I: The technical interest rate used.
$\ddot{a}_{x_r}^{\beta}$: Actual value of a life annuity due of 1 per year, while "x_r" survives, increasing at the accumulative annual rate of β, with I being the technical rate of interest.

The determination of the IRR for each scenario "s" can also be expressed directly with the following equation:

$$\sum_{x=x_e}^{x_r-1}\left[Cr_x \; W_x^s \; {}_{x\text{-}x_e}p_{x_e}\right](1+IRR^s)^{-(x-x_e)} = \sum_{x=x_r}^{\omega}\left[P_{x_r}^s \; {}_{x\text{-}x_e}p_{x_e} \prod_{t=x_r}^{x}(1+\alpha_t^s)\right](1+IRR^s)^{-(x-x_e)} \qquad [7.]$$

The models based on the notional defined contribution account system used for calculating the initial retirement pension and its later variation are the following[16]:

[15] See Appendix for the actuarial evaluation.
[16] The fact that we use different indices in the same model illustrates the importance of our assumption between these factors.

Table 5: Formulae for calculating the initial pension and its later variation[17].			
Model	**Revaluation of the contribution base**	**Notional rate for contributions**	**Notional rate for pensions**
1	RPI	GDP	RPI
2	RPI	AEI	RPI
3	RPI	GDP	RPI±GDP differential
4	RPI	GDP	RPI±AEI differential
5	RPI	AEI	RPI±GDP differential
6	RPI	AEI	RPI±AEI differential
7	RPI	TSSCI	RPI
8	RPI	TSSCI	RPI±TSSCI differential
11	AEI	GDP	RPI
12	AEI	AEI	RPI
13	AEI	GDP	RPI±GDP differential
14	AEI	GDP	RPI±AEI differential
15	AEI	AEI	RPI±GDP differential
16	AEI	AEI	RPI±AEI differential
17	AEI	TSSCI	RPI
18	AEI	TSSCI	RPI±TSSCI differential

When the notional rate for pensions in the formulae in Table 5 shows, for instance, RPI±AEI differential, this means that pensions already in payment will be adjusted according to the RPI plus a positive or negative differential. In this example the differential depends on the behavior of the real AEI for each scenario relative to the expected AEI (average value). If the real AEI is greater than the expected one or the benchmark, then the variation for pensions in payment will be greater than the RPI. If the opposite is true, then it will be less. The other benchmark macroeconomic variables operate in the same way. This system of increasing pensions is inspired by the Swedish experience (see Table 1). The assumption that the correlation between the indices is perfect validates working on one model with different indices.

If the average expected replacement rate is analyzed with each of these models, because a number of them use the same indices to calculate the increase of contributions and the notional rate, the sixteen models analyzed can be broken up into six groups.

The table below shows the replacement rate amounts for each of these groups. For model "m", this replacement rate is obtained with the following formula:

$$\overline{RR}_m = \sum_{s=1}^{20} p^s RR_m^s \qquad [8.]$$

$$RR_m^s = \frac{P_{xr,m}^s}{W_{xr-1,m}^s} \qquad [9.]$$

where:

$\overline{RR}_m$: Average expected replacement rate.

RR_m^s: Expected replacement rate under scenario s.

$P_{x_r,m}^s$: Initial pension expected under scenario "s" for model "m".

$W_{xr-1,m}^s$: Final salary expected under scenario "s" for model "m".

[17] The mathematical application of each of the models is similar to that for the actuarial evaluation explained in the Appendix.

Table 6: Average expected replacement rate. Retirement age 65.		
Average expected replacement rate	**Model**	**Groups**
38.33%	12	1
	15	
	16	
38.32%	11	2
	13	
	14	
38.31%	2	3
	5	
	6	
38.3%	1	4
	3	
	4	
34.17%	17	5
	18	
34.11%	7	6
	8	

The first group, which assumes that the contribution rate increases in line with the AEI and that the capitalization of contributions is also carried out according to the AEI, is the one that generates the highest replacement rate. In a first approach it could be said that, of the models analyzed, those belonging to the first group are those which any well-informed beneficiary would choose *a priori*. Indeed, all those with contribution bases that vary in line with the AEI are preferable, in terms of the average expected replacement rate, to those which follow the RPI. This is because a higher replacement rate is achieved in models where a greater contribution effort is made.

Another thing that attracts the attention is the value of the replacement rate. After forty years of contributing, the formula that provides the best replacement rate is the one for around 38.33%. This is in sharp contrast to the replacement rate currently supplied by the system, which is around 89%. If the notional accounts system were applied, in the best of cases the initial pension would reach 43% of that obtained under the present PAYG system applying in Spain. If people started work at 20 instead of 25, these replacement rates would be slightly higher, reaching 42% in the case of group 1.

The information in Table 6 is valid for carrying out a first comparative analysis between the different models. To analyze beneficiary risk, various moments of the IRR distribution, such as the average, the deviation and the VaR, need to be calculated. In no case can the average replacement rate be a good indicator of the aggregated financial risk since it would only take account of the randomness associated with the "capitalization" of the contributions, leaving aside the randomness associated with the adjustment (increase) of those pensions already in payment.

Table 7: Average IRR and expected deviation for men (M)and women (W). Retirement age 65.							
IRRM average	**Model**	**IRRM deviation**	**% DevM**	**IRRW average**	**Model**	**IRRW deviation**	**% DevW**
0.026435	14	0.011967	45.27%	0.036003	14	0.012629	35.08%
0.026410	6	0.013565	51.36%	0.035972	6	0.014128	39.28%
0.026398	3	0.022436	85%	0.035958	13	0.011131	30.96%
0.026397	13	0.010576	40.06%	0.035957	3	0.022464	62.48%
0.026389	5	0.012175	46.14%	0.035943	5	0.012630	35.14%
0.026366	12	0.009586	36.36%	0.035914	12	0.009840	27.40%
0.026343	11	0.007988	30.32%	0.035895	11	0.008346	23.25%
0.026342	1	0.007990	30.33%	0.035893	1	0.008350	23.26%
0.025969	15	0.017336	66.76%	0.035521	15	0.017981	50.62%
0.025954	16	0.019320	74.44%	0.035515	16	0.020117	56.64%
0.022799	17	0.011203	49.14%	0.032603	18	0.017254	52.92%
0.022798	8	0.016694	73.23%	0.032600	8	0.017258	52.94%
0.022798	18	0.016695	73.23%	0.032580	17	0.011354	34.85%
0.022795	7	0.011208	49.17%	0.032574	7	0.011363	34.88%
0.021118	2	0.006257	29.63%	0.030271	2	0.006276	20.73%
0.019811	4	0.007343	37.06%	0.028943	4	0.007655	26.45%

The results obtained for the average expected IRR are shown separately for men and women in Table 7. Also shown is the percentage of expected deviation from the IRR for each model. Four basic aspects need to be highlighted:

1) The analysis of the average IRR shows clear differences between men and women. This discrepancy comes about because the joint average life expectancy of men and women at retirement age was used when calculating the initial pension. Given that women have a higher life expectancy, the expected return on contributions is much higher.

2) If Tables 6 and 7 are compared, no clear relation between the replacement rate and the IRR can be seen. This is due to the fact that the replacement rate refers exclusively to the initial pension and, in addition, the contribution effort made is not taken into account. The IRR, however, relates all the probable inflows and outflows, and takes into account how the pension can vary over time.

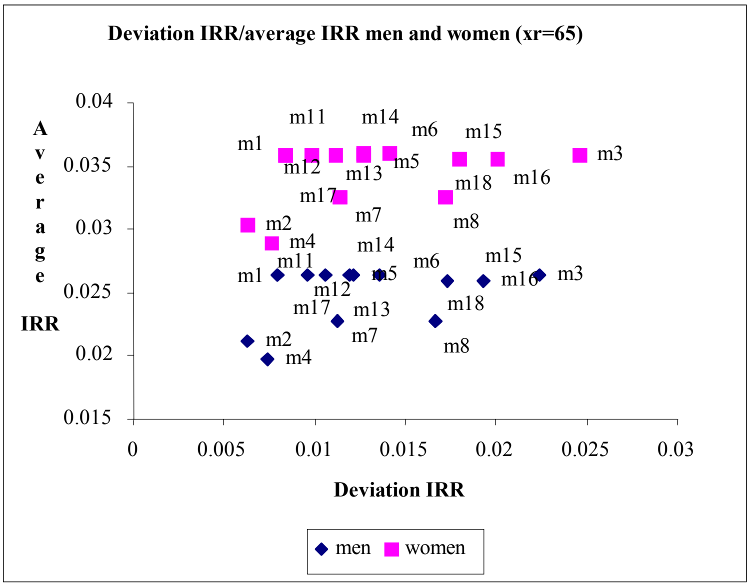
3) There are only very small differences between the real average expected IRR for both men and women in the first ten models. This appears to indicate that the participant-beneficiary could choose any of them using his or her degree of risk aversion as a basis for making the decision.

4) The values obtained for the real IRR appear to be surprisingly low[18], but in fact they are not that low as the calculation is being considered *a priori*. The values will increase proportionally as the contributor is assumed to grow older. Calculating the IRR *a priori* is considered a better way of showing the risk the contributor-beneficiary faces, given that it takes into account the uncertainty associated with the index for adjusting pensions and that for capitalizing contributions.

5) The average values undergo deviations, which implies that those models that generate a greater deviation of the IRR relative to the average IRR are riskier. The listing in order of

[18] With similar assumptions, and assuming current legislation constant for the whole time period considered, the real IRR would be 4.23% and 5.01% for men and women respectively.

deviation is the same for men and women as they depend on the same volatility factors. Model 3 is seen to be the one showing the highest risk in terms of typical deviation, while Model 2 has the least. In general terms, IRR deviation for women is greater than for men. All the above can be seen more clearly in Graph 3.



Graph 3: Relation between average IRR/average deviation for men and women

Looking from the perspective of risk, an interesting instrument to apply is the Value-at-Risk (VaR)[19]. As Jorion (1997) writes, "*VaR summarize the expected maximum loss (or worst loss) over a target horizon within a given confidence interval*"

In the analysis below, this expected loss is taken to be the minimum value of the IRR. For δ % probability, and provided that the conditions included in the scenario generation model used are maintained, the minimum IRR value for each of the models is expressed as:

$$VAR_{\delta}(IRR) = F^{-1}_{IRR^s}(1-\delta) = Sup\left[IRR^s : F_{IRR^s}(IRR^s) \leq (1-\delta)\right] \qquad [10.]$$

where $F^{-1}_{IRR^s}(1-\delta)$ may be seen to be the inverse of the distribution function of the random IRR variable for an accumulated probability of $(1-\delta)$; i.e, the $(1-\delta)$ quartile.

Clearly those alternatives with a lower VaR imply greater risk. It can be seen from Table 8, where $VAR_{0.95}$ is calculated, that the results for men and women coincide in most cases.

[19] As Aragonés and Blanco (1999) point out, the VaR is rapidly becoming a standard for risk management in institutions all over the world. *The greatest advantage of VAR is that it summarizes in a single number, easy to understand, because it offers information about the potential loss that has to be faced during a particular period.* Although the VaR is usually used in business and financial management, in this paper it is applied to analyze beneficiary risk.

Table 8. $VaR_{0.95}$ for expected IRR for men (M) and women (W). Retirement age 65.			
$VaR_{0.95}$ IRR M	**Model**	**$VaR_{0.95}$ IRR W**	**Model**
0.00940	1	0.0181	1
0.00931	11	0.0180	2
0.00884	2	0.0171	11
0.00770	12	0.0159	12
0.00622	3	0.0147	3
0.00620	13	0.0128	4
0.00585	4	0.0124	13
0.00288	5	0.0116	5
0.00123	14	0.0095	14
0.00028	15	0.0089	15
-0.00139	7	0.0080	17
-0.00140	17	0.0079	7
-0.00256	6	0.0058	16
-0.00259	16	0.0058	6
-0.01300	8	-0.0043	8
-0.01321	18	-0.0045	18

For both men and women the models that provide least value are 8 and 18, with 16 therefore being the riskiest. The three models with the least risk for both men and women are 1, 11 and 2.

If the beneficiary is a man who decides to use the TCSSI as a notional rate for contributions and the RPI ± TCSSI differential as a notional rate for pensions, there is 95% chance of his IRR being greater than –1.321%. However, if the same man chooses GDP as a notional rate for contributions and RPI as a notional rate for pensions, there is a 95% probability that his IRR could be greater than 0.94%.

In the Model 1 supposition, the VaR means a 64% reduction in average expected IRR, and between 58% and 40% in Model 2 for men and women respectively. The men's VaR is lower than the women's, and so women therefore run less risk.

IV.1-Beneficiary risk and risk aversion

In order to carry out an overall risk analysis, the beneficiary's subjectivity in evaluating risk through his risk aversion must be introduced. If the risk analysis were made in terms of the expected utility of the IRR, difficulties could arise due to the fact that on the one hand the IRR may take on negative values whose utility could be difficult to define, and on the other because its relation to the beneficiary's level of consumption is not direct. Levy and Markowitz (1979) and Kroll, Levy and Markowitz (1984) show that the expected utility of the return can be estimated via a function that relates the average and the variance. This function will reflect the attitude to risk . Thus, if the function used quantifies the beneficiary's attitude to risk, the choice of model obtained will follow this criterion or value, the beneficiary opting for riskier models when the function reflects that he is less averse to risk, and more conservative models the more averse to risk he is.

The function used, based on Markowitz's theory, is as follows:

$$JR(IRR) = \mu_{IRR} - \gamma\ \sigma_{IRR} \qquad [11.]$$

where:

μ_{IRR} : average value of the IRR

σ_{IRR} : typical deviation of the IRR

γ : parameter that quantifies risk aversion.

If $\gamma = 0$, the individual is neutral to risk.

If $\gamma > 0$, the individual is averse to risk. The higher γ is, the greater the risk aversion will be.

The beneficiary will choose whichever model supplies the greatest value for this function, in relation to his risk aversion. The option chosen by an individual who is neutral to risk coincides with the maximization of the average IRR.

Different risk aversion coefficients are assumed, classified as shown in Tables 9 and 10. Those beneficiaries with greater risk aversion (0.5), both men and women, choose model 11 for preference. This model is one of those that represent less risk in VaR terms, and the combination of average IRR and deviation of IRR represents the second lowest risk.

With a low risk aversion coefficient (0.01), the optimal choice is similar to the neutral case.

Table 9: Classification of formulae for men according to their aversion to risk. Retirement age 65.

γ =0.5	γ =0.1	γ =0.05	γ =0.01	γ =0 neutral
11	11	11	14	14
1	1	1	13	6
12	12	12	6	3
13	13	13	12	13
14	14	14	5	5
5	5	5	11	12
6	6	6	1	11
2	15	15	3	1
15	16	16	15	15
17	3	3	16	16
7	17	17	17	17
16	7	7	7	8
4	8	8	8	18
8	18	18	18	7
18	2	2	2	2
3	4	4	4	4

Table 10: Classification of formulae for women according to their aversion to risk. Retirement age 65.				
γ =0.5	γ =0.1	γ =0.05	γ =0.01	γ =0 neutral
11	11	11	14	14
1	1	1	13	6
12	12	12	6	3
13	13	13	12	13
14	14	14	5	5
5	5	5	11	12
6	6	6	1	11
2	15	15	3	1
15	16	16	15	15
17	3	3	16	16
7	17	17	17	17
16	7	7	7	8
4	8	8	8	18
8	18	18	18	7
18	2	2	2	2
3	4	4	4	4

The results for women are almost identical to those for men, as can be seen in Table 10.

IV.2-Beneficiary risk and retirement age

One of the theoretical advantages of the notional accounts system is that it more directly reflects the individual's preferences with regard to the pension they wish to receive at the end of their working life, since the system manages to tighten the pension-contribution relationship, and therefore achieves greater equality or "actuarial fairness". It is interesting to study whether retiring earlier or later than the accepted benchmark age of retirement would have any important effect on the risk faced *a priori* by the beneficiary. With this end in view two suppositions are made:

1) The beneficiary takes early retirement at 60.
2) The beneficiary defers retirement age until 70.

Table 11 shows the results for the replacement rate when the beneficiary retires at 65, and the reduction coefficient for each of the suppositions put forward. In this table the models are ordered by the numerical value of the replacement rate, from highest to lowest. If the beneficiary retires before 65, the reduction coefficient is simply rate 60 divided by rate 65. If the coefficient is less than one it implies a penalty because the initial pension is decreased, whereas if it is greater than one the effect is reversed.

The adjustment coefficient is less than one when the beneficiary takes early retirement at 60 and greater than one when retirement age is deferred until 70.

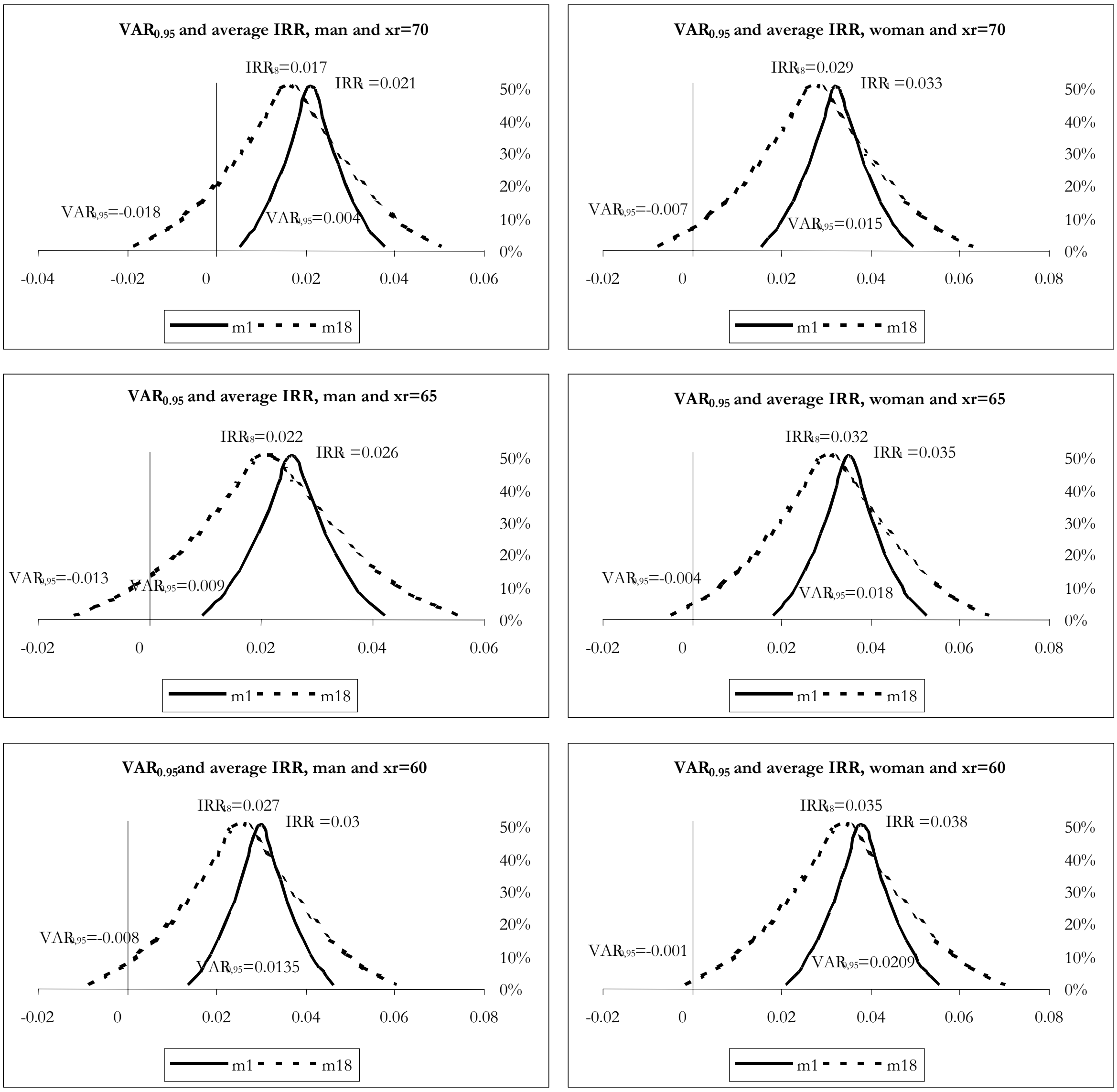
The replacement rate increases with age and the number of years contributions have been paid. The disincentive to work brought about in a badly designed defined benefit PAYG system therefore seems to be mitigated.

The cancellation or weakening of this disincentive to work is questionable if the data for the average expected IRR are analyzed. The results of this are shown in Table 4, which includes the VaR $_{0.95}$ and average IRR for the riskiest (Model 18) and least risky (Model 1) models for both men and women and different retirement ages. It can be seen that for both men and women the average IRR decreases with age and the number of years contributions have been paid. This clearly indicates that the cancellation of the work disincentive effect is much more apparent than real. Furthermore, from the point of view of the expected return on contributions, any of the formulae tested would bring about the opposite effect to that expected.

Table 11: Average expected replacement rate for x_r=65 and adjustment coefficient for x_r=60 and x_r= 70.

60 years		Model	65 years	Model	70 years		Model
0.82	31.46%	11	38.33%	12	1.28	49.30%	12
		13		15			15
		14		16			16
0.82	31.45%	1	38.32%	11	1.28	49.19%	2
		3		13			5
		4		14			6
0.78	30%	12	38.31%	2	1.23	47.22%	11
		15		5			13
		16		6			14
0.78	29.96%	2	38.30%	1	1.23	47.11%	1
		5		3			3
		6		4			4
0.83	25.83%	17	34.17%	17	1.21	41.44%	17
		18		18			18
0.83	28.52%	7	34.11%	7	1.21	41.34%	7
		8		8			8

In terms of risk as measured from the perspective of the $VaR_{0.95}$, it would not appear to be a good decision *a priori* to delay the age of retirement and pay contributions over more years either, since the expected result is unfavourable for both men and women. It could be said that it is riskier to retire at 70 than at 60, since the extreme IRR values are lower at 70 than at 60. If it were considered, as Valdés-Prieto (2002) suggests, that by taking early retirement the retiree has more leisure time - which is valued very positively - then the conclusions reached would be strengthened even more in the sense that taking early retirement would imply a greater expected return, less risk to bear and greater leisure opportunities.



Graph 4. VaR $_{0.95}$ and average IRR for the riskiest (m18) and least risky (m1) models, for men and women and different retirement ages.

Another interesting aspect (see Table 12) is that taking the degree of risk aversion combined with the envisaged retirement age also affects the *a priori* choice of model of notional retirement formula, although this would have a greater impact on those individuals who are less averse to risk.

Table 12: Preferred models according to sex, retirement age and degree of risk aversion.

Sex	Retirement age	γ =0.5	γ = 0.1	γ = 0.05	γ = 0.01	γ = 0 neutral
Men	60	11	11	11	14	14
	65	11	11	11	14	14
	70	12	12	12	12	12
Women	60	11	11	11	14	14
	65	11	11	11	14	14
	70	12	12	12	12	12

In fact one of the criticisms usually made of notional accounts systems, as pointed out by Disney (1999), is that the contributors take on the risk of the evolution of the index and are subject to a risk-return trade-off they have not chosen, i.e. their aversion to risk is not taken into account like it is in private capitalization funds. One way of avoiding this problem would be to have a menu of retirement formulae available, like those proposed for Spain. Then every three or four years or so the contributor could change the contribution variation index according to his perception of risk and the evolution and envisaged path of the indices.

Finally, the model and retirement age that offer the least risk for an 25-year-old, based on his risk aversion coefficient, are those that appear in Table 13.

Table 13: Preferred models and retirement age according to sex and degree of risk aversion.

Sex	$\gamma = 0.5$	$\gamma = 0.1$	$\gamma = 0.05$	$\gamma = 0.01$
Men	11 (60 years)	11 (60 years)	11 (60 years)	14 (60 years)
Women	11 (60 years)	11 (60 years)	11 (60 years)	14 (60 years)

V.-CONCLUSIONS

An unexplored aspect of notional defined contribution accounts is the study and quantification of the risk faced by the contributor-beneficiary. In this paper we have estimated the expected average IRR and analyzed the risk involved in sixteen retirement formulae based on notional philosophy to which the beneficiary would be exposed if, in Spain in 2003, it were decided to introduce a retirement pension system based on notional accounts.

The *a priori* average expected IRR for both men and women following any of the formulae tested based on representative indices of relevant macroeconomic variables is quite clearly lower than the IRR awarded today on contributory retirement pensions by current Spanish legislation. The envisaged replacement rate in the most favourable formula barely reaches 43% of that obtained today. This only goes to highlight the profound structural actuarial imbalance present in the current configuration of the defined benefit retirement pension system in Spain.

The models preferred for both male and female beneficiaries who are neutral to risk are 14 and 6, in descending order. The first of these capitalizes the contributions in line with the expected evolution of the GDP; the second follows the AEI. In both cases the pensions can participate in the probable upward fluctuations of the salaries index above that foreseen.

Taking the degree of risk aversion into account slightly changes the preferences established by following only the criterion of average expected IRR. Individuals with a more marked aversion to risk, both men and women, would always choose model 11 first, opting next for model 14 according to how their attitude to risk tends towards neutral.

From the point of view of risk to be borne as measured by the VaR, the three models proposed that have the lowest value and therefore less risk are 1, 11 and 2, as far as men are concerned, and 1, 2 and 11 for women. Models 1 and 11 use the expected GDP and RPI as a notional rate for contributions and pensions in payment respectively, while model 2 uses the AEI for the capitalization of contributions. In any case, the minimum the minimum level of IRR in the best model (m1), 0.94% and 1.81% for men and women respectively, would be substantially less than the IRR stemming from current legislation, 4.23% and 5.01% for men and women respectively, under the highly improbable supposition that the system could be maintained without changes in regulations for the whole of the time period of the projection.

The effect of deferring retirement age, although providing a higher expected replacement rate for all the models studied and an adjustment coefficient greater than one, seems to corroborate that these systems really do provide a disincentive to leave the labor market. At the same time, other effects of deferring retirement age are a lower expected IRR and a greater risk be faced. This would suggest that the possible introduction in Spain of any of the models studied would need some sort of correctional element in order for it not to bring about the opposite effect to that desired in the form of growing risks and decreasing returns.

Finally, if a notional accounts system were introduced, it would be best if contributors were able to change regularly the contributions notional index, like in individual capitalization account systems, so as to adjust it to their perception of risk and to the predicted evolution and path of the indices.

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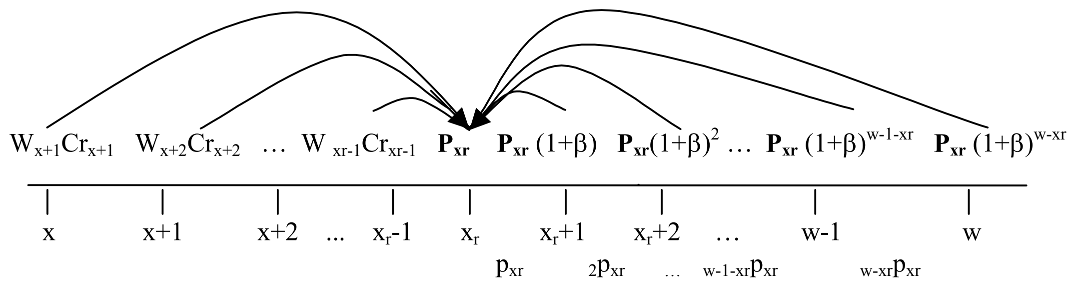
VII.-APPENDIX I: ACTUARIAL EVALUATION FOR DEFINED CONTRIBUTION NOTIONAL ACCOUNTS

The approach follows the assumption that the contributor has actually reached retirement age, and therefore the contributions are made in reality and the pension will be received by him every year if he survives.

$W_t \cdot Cr_t$ denotes the contributions made by the individual at age t. These are valued at a particular rate at age x_r (time of retirement). The total contributions give entitlement to an indexed lifetime annuity that the individual will receive during his retirement, with the initial



pension being P_{xr}. At moment x_r the value of the actuarial pension is calculated by matching the contributions made during working life to future benefits. In this way the equation fulfills financial and actuarial principles.

Following the above procedure, the general formula for calculating the pension will be obtained by matching, at moment "x_r", the accumulated notional fund (K) with the current actuarial value of the expected pension due:

$$\overbrace{\sum_{t=x_e}^{x_r-1} Cr_t \cdot W_t \prod_{i=t}^{x_r-1}(1+r_i)}^{K} = P_{x_r} \underbrace{\sum_{t=x_r}^{w} \frac{(1+\beta)^{t-x_r}}{[(1+I)]^{t-x_r}} \, {}_{t-x_r}p_{x_r}}_{G=\frac{1}{g}} = P_{x_r} \, \ddot{a}_{x_r}^{\beta} = P_{x_r} \frac{1}{g} \qquad [12.]$$

where:

x_e : Age of the individual on entering the labor market.
Cr_t: Contribution rate at moment t,
W_t: Salary or contribution base at moment t,
I: The technical interest rate used.

r: Annual index used to capitalize the contributions,
r_i: Annual index used to capitalize the contributions during period i,
β: Annual index used to determine the initial pension,
P_{xr}: Pension at retirement age "x_r",
${}_{t-xr}p_{xr}$: Probability that an individual aged "x_r" will reach age "t", or will live "t-x_r" more years.
$\ddot{a}^{\beta}_{x_r}$: Actual value of a life annuity due of 1 per year, while "x_r" survives, increasing at the accumulative annual rate of β, with I being the technical interest rate used.)
K: Accumulated notional account.
G: Inverse of the conversion factor.
g: Conversion factor.

The unknown factor of Equation 13 is the amount of pension that the worker will receive at the moment of retirement. This is because the later amounts are obtained by adjusting the initial pension in line with the index chosen. If we find the value of the pension at retirement age:

$$P_{x_r} = \frac{K}{G} = g\,K = g \sum_{t=x}^{x_r-1} Cr_t \cdot W_t \prod_{i=t}^{x_r-1}(1+r_i) \qquad [13.]$$

where:
g: g-value, predetermined conversion factor, which is equal to the inverse of the actuarial pension defined previously:

$$g = \frac{1}{\sum_{t=x_r}^{w} \frac{(1+\beta)^{t-x_r}}{[(1+I)]^{t-x_r}} \; {}_{t-x_r}p_{x_r}} = \frac{1}{\ddot{a}^{\beta}_{x_r}} \qquad [14.]$$

On the other hand, if pension adjusting policy were designed in such a way that there was perfect indexation of pensions to the growth rate of the relevant variable, (1+β) = (1+I), then the discount factor is equal to the unit, and so the conversion factor becomes the inverse of life expectancy at retirement age plus the unit:

$$g = \frac{1}{\sum_{t=x_r}^{w} {}_{t-x_r}p_{x_r}} = \frac{1}{1+e_{x_r}} \qquad [15.]$$

This demographic parameter would therefore appear explicitly in this extraordinarily transparent pension formula:

$$P_{xr} = \frac{\sum_{t=x}^{x_r-1} Cr_t \cdot W_t \prod_{i=t}^{x_r-1}(1+r_i)}{1+e_{x_r}} = \frac{K}{G} \qquad [16.]$$