EUROPEAN VOLATILITY TRANSMISSION WITH STRUCTURAL CHANGES IN VARIANCE

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ABSTRACT

The aim of this study is to analyze the European transmission of information, through volatilities with structural changes in variance, for the stock indexes of principal European stock markets: UK, Germany, France, Italy and Spain, for European Union and Swiss as European zone no euro

Our most significant contributions to this area of study are, firstly, the influence that the consideration of structural changes in the variance has in relation to the interaction between the main European stock indexes, through the study of volatility transmission.

In order to include structural changes in variance, we followed the methodology put forward by Sansó et al. (2002), which detects these changes endogenously. Although numerous studies have investigated the volatility spillover among international stock markets, this aspect has not been considered. Our results show the importance of taking them into consideration since they influence the schema of information transmission between markets.

Secondly, the use of bivariate asymmetric GARCH models which allow for variation in correlation over time. This model has obvious advantages: market interactions can be investigated and analyzed in a one step estimation procedure, thus eliminating the need to use estimated regressors. Additionally, the hypothesis that innovations within and across markets influence volatility asymmetrically can be explicitly tested. Finally, we verify the effect of information between Spain versus the main European stock markets.

Keywords: European volatility transmission, time-varying covariance, international markets, bivariate GARCH, asymmetry, structural changes in variance.

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1.- Introduction

The aim of this study is to analyze the European transmission of information, through volatilities with structural changes in variance, among the most important markets included in European area, specifically Spanish versus German; United Kingdom, France, and Swiss stock indexes calculated by datastream, were used.

Research into volatility transmission has found evidence that the higher the globalization of the economy, the greater the incidence of transmission, thereby providing grounds for the idea that volatility transmission between financial markets could be due to the information generating process, in line with the models put forward by Ito et. al (1992) for the foreign exchange market. The works of Hamao et al. (1990), Bae and Karolyi (1994), Karolyi (1995), Koutmos and Booth (1995), Karolyi and Stulz (1996), Koutmos (1996), Song et al. (1998) and Antoniou et al. (2001) should be mentioned as the most noteworthy studies on information transmission via volatility between international stock markets.

Most studies on volatility transmission use either univariate Garch models that include exogeneous variables representing the variance or innovations of another market (Hamao et al., 1990, Kim and Rogers, 1995) in a two-step procedure, or multivariate GARCH models where it is assumed that the correlation coefficient between stock returns is constant (Antoniou et al., 2001, Theodossiou and Lee, 1993, Koutmos and Booth, 1995, Koutmos, 1996; Kanas, 1998).

One aspect that has not been considered in the above-mentioned studies is the existence of structural changes in the modeling of the variance in the different markets analyzed. This may give rise to biased results deriving from a poor specification of the conditional variance used (Lamoreux and Lastrapes, 1990). Furthermore, consideration of these changes will enable us to differentiate between distinct variance regimes and to analyze how they explicitly influence the information transmission process.

Our most significant contributions to this area of study are, firstly, the influence that the consideration of structural changes in the variance has in relation to the interaction between the main European stock indexes, through the study of volatility transmission. The methodology devised by Sansó et al. (2002) is proposed to carry this out.

Secondly, the use of bivariate asymmetric GARCH models which allow for variation in correlation over time. This model has obvious advantages: market interactions can be investigated and analyzed in a one step estimation procedure, thus eliminating the need to use estimated regressors. Additionally, the hypothesis that innovations within and across markets influence volatility asymmetrically can be explicitly tested (Koutmos, 1996). Finally, we verify the effect of information between Spain versus the main European stock markets.

Various explanations can be found for volatility transmission within the financial literature. The first is based on the APT model developed by Ross (1976) which points to the existence of common factors that affect the valuation of assets in various markets as the cause of volatility transmission. The work of King et al. (1994) can also be included in this line of thought, which assumes there to be both observable and non-observable factors, although they conclude that volatility accounted for by observable
factors is very small.

The second possibility is that the arrival of information on the market comes in waves and causes volatility as it is incorporated into the price. In other words, it is the dynamics of the system itself which induces volatility persistence, as postulated in the market microstructure models developed by Kyle (1985) and Admati and Pfleiderer (1988). The asymmetry of information leads those in possession of this information to slow down its incorporation for the sake of obtaining higher returns, which would explain the persistence of volatility, as expounded by Ederington and Lee (1995) and Donders and Vorst (1996).

The rest of the paper is organized as follows: Section 2 describes the data and methodology employed. Section 3 presents the empirical results and Section 4 contains the concluding comments.

2.- Data and methodology

The data used came from the daily closing prices provided by Datastream for the stock indexes of principal European stock markets: UK, Germany, France, Italy and Spain, for European Union and Swiss as European zone no euro. The essential advantage of this database is that all the indexes are homogeneously presented and all use the same currency, the dollar. They represent about 80% of the effective volume of the markets considered.

We analyzed the period from 1/10/1995 to 12/31/2000, which gave us 1560 observations. The series of daily returns were calculated as the difference between the logarithm of the closing prices between two consecutive sessions:

\[ R_{i,t} = 100 \times \log \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \]

Table 1 summarizes the descriptive statistics of stock returns: the mean and the standard deviation of the rate of return for each market during the sample period.

The measures for skewness and kurtosis indicate that the distributions or returns for all three markets are negatively skewed and leptokurtic relative to the normal distribution. The Jarque-Bera (JB) statistic rejects normality at any level of statistical significance in all cases. The Ljung-Box statistic for up to 15 lags, calculated for both the return and the squared return series, indicates the presence of significant linear and non-linear dependencies in the returns of all markets. As Stoll and Whaley (1990) indicate, linear dependence may be due to non-synchronous trading of the stock that makes up each index. Nonlinear dependence can be captured satisfactorily by ARCH models (Bollerslev et al. 1994).

Many financial series show a high degree of kurtosis as well as volatility clustering, in such a way that large changes in prices are followed by periods of large changes in any sign, while small changes tend to be followed by small changes. Bollersev (1986) proposed the use of GARCH models to gather these facts. In most empirical studies of

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1 The database was provided by Jorge Mina of Datastream-Spain, for whose collaboration we are most grateful.
financial series where this type of model has been used, a high degree of persistence in variance is detected\(^2\).

Lamoreux and Lastrapes (1990) point out that the high degree of persistence is due to the poor specification of the model, particularly to the fact that possible deterministic changes in the unconditional variance of these series is not taken into consideration. In order to do this, they propose that dummy variables representing these changes be included in the GARCH model employed. They conclude that in this way, the degree of persistence obtained is considerably reduced. However, as these authors themselves point out, the determination of the moment, together with the duration of the changes, is arbitrarily carried out.

In order to detect these possible changes in variance, we propose a methodology based on the iterated cumulative sum of squares of the series analyzed (Sansó et al. 2002) that modifies the test (ICCS) proposed by Inclan and Tiao (1994)\(^3\). Sansó et al., take into account the fourth moment properties of the disturbances and the conditional heteroskedasticity presented in almost all financial time series. This methodology allows us to determine the number and temporal moment in which the changes occur, together with their size and duration, thereby avoiding the above-mentioned problem of arbitrariness.

Table II shows the total number of variance changes detected and the opening and closing dates of the period studied for each of the three Stock Exchanges analyzed, and the unconditional variance associated with each of these periods. Graphs appendix show the returns on the Spanish, France, Swiss, UK, and Germany. markets together with the representation of the variance regimes detected.

In order to estimate the bivariate GARCH models, the BEKK model (Engle and Kroner, 1995) is proposed. A bivariate GARCH model can be characterized by the following expressions. Expression (1) shows the first order moments expressed according to a bivariate VAR model (Engle and Granger, 1987)\(^4\).

\[
R_t = \alpha + \sum_{i=1}^{p} \beta_i^R R_{i,t-1} + e_t \tag{1}
\]

\[
e_t / \Omega_{e_t} \sim \mathcal{N}(0, H_t) \tag{2}
\]

\[
H_t = C + B H_{e_{t-1}} B + A e_{e_{t-1}} A + G \eta_{t} \eta_{t} \tag{3}
\]

Where \( R_t = (R_{i,t}, R_{j,t}) \) is a vector of a pair of stock exchange returns; \( e_t = (e_{i,t}, e_{j,t}) \) is assumed to follow a normal bivariate distribution with a mean of 0 and variance \( H_t \), where \( \Omega_{e_{t-1}} \) is the information set in t-1. \( H_t \) is the NxN conditional matrix of variance-covariance; and \( C, A, B \) and \( G \) are N×N matrices of parameters with \( C \) symmetric and

\(^2\) The high degree of persistence has obvious economic implications, deriving from the effect that it has on the predictability of future value, as shown, for example, by Poterba and Summers (1986).

\(^3\) Previous studies that have also used this methodology include those of Wilson \textit{et al}. (1996) and Aggarwal, \textit{et al}. (1999) who detect changes in variance in various stock exchanges.

\(^4\) We do not include the error correction term since there are no cointegration relationships between the returns series, following the methodology of Johansen and Juselius (1988,1990).
positive definite. Moreover, to consider the asymmetrical response of the volatility to news of a different sign, we used the extension proposed by Engle and Ng (1998) of the asymmetric BEKK model including $\eta_t = \min(0, e_t)$ (Gagnon and Lypny, (1995); Kroner and Ng, (1998)). This model allows us to study the volatility transmission between both markets as well as the signs of the shocks, when the innovations are of both a positive and negative character.

For each model, the Akaike, Bayesian and Hannan-Quinm information criteria were employed to determine the conditional variance lag lengths, which suggests that a BEKK-GARCH(1,1) specification is appropriate for the conditional variance. Additionally, in order to take account for the changes in the variance, matrix $C$ incorporates the changes detected in the variance, with $D_i$ and $D_j$ as the dummy variables which show the various regime changes found; $B_i$ and $B_j$ are the parameters which accompany these variables. The complete model for the variance-covariance matrix is covered in expression (3).

$$
\begin{bmatrix}
    h_{11,t} & h_{12,t} \\
    h_{12,t} & h_{22,t}
\end{bmatrix} = \begin{bmatrix}
    \sum_{i=1}^{n} B_{1i} C_{1i,t} & C_{12} \\
    C_{12} & \sum_{j=1}^{m} B_{2j} C_{2j,t}
\end{bmatrix} + 
\begin{bmatrix}
    A_{11} & A_{12} \\
    A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
    e_{1,t-1}^2 & e_{1,t-1} e_{2,t-1} \\
    e_{2,t-1} & e_{2,t-1}^2
\end{bmatrix} + 
\begin{bmatrix}
    B_{11} & B_{12} \\
    B_{21} & B_{22}
\end{bmatrix} \begin{bmatrix}
    h_{11,t-1} & h_{12,t-1} \\
    h_{12,t-1} & h_{22,t-1}
\end{bmatrix} + 
\begin{bmatrix}
    D_{11} & D_{12} \\
    D_{21} & D_{22}
\end{bmatrix} \begin{bmatrix}
    \eta_{1,t-1}^2 & \eta_{1,t-1} \eta_{2,t-1} \\
    \eta_{2,t-1} \eta_{1,t-1} & \eta_{2,t-1}^2
\end{bmatrix} 
$$

(3)

To take into account the intense cross-correlation between $e_{i,t}$ and $e_{j,t}$, the information from one market ($e_{i,i}$) that is not in the other ($e_{j,i}$) has been included in the variance equations. In this sense, $e_{i,t}$ ($e_{j,i}$) is orthogonal to $e_{j,t}$ ($e_{i,i}$). Moreover, these equations permit these innovations ($e_{i,t}$ and $e_{i,i}$) to influence the conditional volatility asymmetrically, as do their own innovations ($e_{i,t}$ and $e_{j,t}$).

Expression (4) shows the development of the conditional variance, and incorporates the orthogonalized innovations and the changes of variance for hypothetical market R1.

$$
H_{11,t} = \sum_{i=1}^{n} B_{1i} C_{1i,t} + A_{11}^2 e_{i,t-1}^2 + 2A_{11} A_{21} e_{1,t-1} e_{2,t-1} + A_{21}^2 e_{2,t-1}^2 + 
B_{11}^2 H_{11,t-1} + 2B_{11} B_{21} H_{12,t-1} + B_{21}^2 H_{22,t-1} + 
D_{11}^2 \eta_{1,t-1}^2 + 2D_{11} D_{21} \eta_{1,t-1} \eta_{2,t-1} + D_{21}^2 \eta_{2,t-1}^2 
$$

(4)

5 For the modeling of matrix $e$, the orthogonalized errors of the dependent stock exchange were taken.
6 $e_{i,t}$ is computed as the residuals from the following regression $e_{i,t} = k_0 + k_1 e_{j,t} + e_{i,t}$.
Where $\eta_t = \text{Min}(0, \epsilon_t)$.

Irrespective of the modeling of the conditional moments, and supposing the normality in the disturbances ($\epsilon_t$), the logarithm of the likelihood function for a sample of $T$ observations will be represented by expression (5), where $\Theta$ is the set of all the parameters to be estimated both of the returns and of the conditional variance:

$$L(\Theta) = -T \log 2\pi - 0.5 \sum_{t=1}^{T} \log|H_t(\Theta)| - 0.5 \sum_{t=1}^{T} \epsilon_t(\Theta)'H_t^{-1}(\Theta)\epsilon_t(\Theta)$$  (5)

The algorithm proposed by Berndt et al. (BHHH) (1974) was used to estimate the set of parameters ($\Theta$).

3.- Empirical results

A summary of the results obtained both with and without the changes in variance taken into consideration is provided in Table III. The estimated conditional variances with associated p-values (parenthesis) and likelihood function values (L) for the three stock markets returns are presented in this table.

The first result we consider worthy of mention is that the inclusion of the dummy variables representative of various variance regimes have influence on the conclusions referring to the structure of the transmission of information. In this way, to stand out for example, to UK and France $A_{12}$ will no longer be significant when these changes are considered. To Italy, the same effect occur for the $A_{21}$ parameter. In the case of the Swiss markets, the opposite situation arises and $A_{12}$, $A_{21}$ and $B_{12}$ becomes significant.

The LR$^8$ test associated with the null hypothesis of no changes in variance are such that the null can be rejected. This test value (LR test) is included in Table III for each model.

In accordance with the previously outlined results, we focus on presenting the main results obtained by considering the model with changes of variance.

Moreover, caution must be taken when considering the interpretations which could arise from the analysis of the estimation of the results of the parameters, given the modeling used, since the interpretation of the coefficients cannot be directly carried out.

As far as the volatility modeling is concerned, we should differentiate between the effects of the innovations, the variances and the asymmetry (matrixes A, B and D respectively). In the first term, with respect to innovations, we are able to verify that a bidirectional cross-market spillover transmission only exists between Spain-Swiss (coefficients $A_{12}$ and $A_{21}$ are significant). From Germany, UK, France versus Spain

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7 Although the hypothesis of normality is not proven, the parameters can be estimated by maximizing $\Theta$. In this case, estimators of quasi-maximum likelihood are obtained. Weiss (1986) studies the asymptotic properties of these estimators and points out that the estimation of $\Theta$ will be consistent if certain conditions are fulfilled, amongst which are that the first and second order conditional moments are well specified, and that robust standard errors are estimated.

8 The Likelihood ratio statistic calculated as $LR=2\left[L(\Theta_1) - L(\Theta_0)\right]$ where $L(\Theta_1)$ and $L(\Theta_0)$ are the maximum log likelihood values obtained from the GARCH with and without changes models respectively. This statistic is asymptotically $\chi^2$ distributed with degrees of freedom equal to the number of restrictions from the more general model (with changes) to the more parsimonious (without changes)
there is a unidirectional relationship (coefficients $A_{21}$ is significant). Spain influence unidirectionally to Italy ($A_{12}$ are significant).

The patterns in the conditional variance coefficients are not different for all the markets considered. The schema of volatility transmission when the lagged variances are analyzed is bidirectional for all the cases (coefficients $B_{21}$ and $B_{12}$ are significant respectively).

The results of the D asymmetries matrix indicate that the negative innovations of the markets increase the volatility in the same market (coefficients $D_{11}$ and $D_{22}$). The volatility transmission mechanism is asymmetric. Negative innovations in the first market increase volatility in the second market considerably more than positive innovations. This result occurs bidirectionally for Swiss, Italy, France and Germany (France and Germany at 10% significance level), unidirectionally only from Spain versus UK (coefficient $D_{12}$).

4.- Conclusions

This paper analyzes the dynamics and transmission of conditional volatilities with structural changes in variance, across European stock markets, employing the multivariate GARCH models that allow the conditional matrix to be time-varying.

In order to include structural changes in variance, we followed the methodology put forward by Sansó et al. (2002), which detects these changes endogenously. Although numerous studies have investigated the volatility spillover among international stock markets, this aspect has not been considered. Our results show the importance of taking them into consideration since they influence the schema of information transmission between markets.

We can summarize as follows:

There is a bidirectional transmission between Spain and Swiss. From Germany, UK, France versus Spain there is a unidirectional relationship and Spain influence unidirectionally to Italy.

The schema of volatility transmission when the lagged variances are analyzed is bidirectional for all the cases.

The volatility transmission mechanism is asymmetric. Negative innovations in the first market increase volatility in the second market considerably more than positive innovations. This result occurs bidirectionally for Swiss, Italy, France and Germany and unidirectionally only from Spain versus UK.

In general these findings suggest that the markets react not only to local news but also to news originating in the other markets, especially when the news is adverse and that the consideration of structural change is feature to considered in this type of research.

References


Ito, T., Engle, R., Lin, W. 1992: “Where does the meteor shower come from?. The role of


### Table I: Main Statistics of daily Index Returns

<table>
<thead>
<tr>
<th></th>
<th>Spain</th>
<th>UK</th>
<th>Switzerland</th>
<th>Italy</th>
<th>Germany</th>
<th>France</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0706</td>
<td>0.0462</td>
<td>0.0701</td>
<td>0.0665</td>
<td>0.0584</td>
<td>0.0760</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.01197</td>
<td>0.0088</td>
<td>0.0099</td>
<td>0.0134</td>
<td>0.0115</td>
<td>0.0113</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.4696</td>
<td>-0.2408</td>
<td>-0.4337</td>
<td>-0.0931</td>
<td>-0.5122</td>
<td>-0.2796</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>6.9193</td>
<td>4.5985</td>
<td>7.1851</td>
<td>5.3345</td>
<td>6.1058</td>
<td>5.1842</td>
</tr>
<tr>
<td>J-B</td>
<td>1055.11</td>
<td>181.05</td>
<td>1186.63</td>
<td>356.23</td>
<td>694.79</td>
<td>330.21</td>
</tr>
<tr>
<td>(Prob.)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Q(15)</td>
<td>0.048</td>
<td>0.018</td>
<td>0.039</td>
<td>0.055</td>
<td>0.046</td>
<td>0.048</td>
</tr>
<tr>
<td>(Prob.)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Q2(15)</td>
<td>0.146</td>
<td>0.127</td>
<td>0.156</td>
<td>0.133</td>
<td>0.070</td>
<td>0.084</td>
</tr>
<tr>
<td>(Prob.)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

Notes: Stock returns are logarithmic percentage changes. The sample period used is from January 2, 1995 to December 30, 2000. Q(15) and Q2(15) are 15-th-lag Ljung-Box test statistics applied to the original and squared standardized for residuals. The p-values appear in parenthesis.

### Table II Detected number of structural changes in variance with Sansó et al. (2002) methodology.

<table>
<thead>
<tr>
<th>Market</th>
<th>Change</th>
<th>Start period</th>
<th>End Period</th>
<th>Days</th>
<th>Var * 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>1</td>
<td>10/01/95</td>
<td>06/12/96</td>
<td>696</td>
<td>0,0539</td>
</tr>
<tr>
<td></td>
<td></td>
<td>09/12/96</td>
<td>29/12/00</td>
<td>1481</td>
<td>1,1854</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10/01/95</td>
<td>04/12/96</td>
<td>694</td>
<td>0,0266</td>
</tr>
<tr>
<td></td>
<td></td>
<td>05/12/96</td>
<td>20/07/98</td>
<td>592</td>
<td>0,0653</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21/07/98</td>
<td>21/01/99</td>
<td>184</td>
<td>2,280</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22/01/99</td>
<td>29/12/00</td>
<td>707</td>
<td>0,1001</td>
</tr>
<tr>
<td>Swiss</td>
<td>1</td>
<td>10/01/95</td>
<td>10/07/96</td>
<td>547</td>
<td>0,0319</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11/07/96</td>
<td>29/12/00</td>
<td>1632</td>
<td>0,1216</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10/01/95</td>
<td>11/09/97</td>
<td>975</td>
<td>0,1195</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12/09/97</td>
<td>21/01/99</td>
<td>496</td>
<td>0,3551</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22/01/97</td>
<td>29/12/00</td>
<td>707</td>
<td>0,1451</td>
</tr>
<tr>
<td>Germany</td>
<td>1</td>
<td>10/01/95</td>
<td>29/11/96</td>
<td>689</td>
<td>0,0626</td>
</tr>
<tr>
<td></td>
<td></td>
<td>02/12/96</td>
<td>29/12/00</td>
<td>1488</td>
<td>0,1710</td>
</tr>
<tr>
<td>France</td>
<td>1</td>
<td>10/01/95</td>
<td>20/05/97</td>
<td>861</td>
<td>0,0626</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21/05/97</td>
<td>29/12/00</td>
<td>1318</td>
<td>0,1710</td>
</tr>
</tbody>
</table>
### TABLA III European volatility transmission with structural changes in variance

**Spain vs. another Stock markets**

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>UK</th>
<th>France</th>
<th>Italy</th>
<th>Swiss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No dummies</td>
<td>With dummies</td>
<td>No dummies</td>
<td>With dummies</td>
<td>No dummies</td>
</tr>
<tr>
<td>A11</td>
<td>0.222 (0.000)</td>
<td>-0.212 (0.000)</td>
<td>-0.167 (0.039)</td>
<td>0.020 (0.299)</td>
<td>0.009 (0.821)</td>
</tr>
<tr>
<td>A12</td>
<td>-0.069 (0.163)</td>
<td>0.051 (0.369)</td>
<td>-0.072 (0.000)</td>
<td>-0.029 (0.595)</td>
<td>-0.129 (0.007)</td>
</tr>
<tr>
<td>A21</td>
<td>0.181 (0.000)</td>
<td>-0.135 (0.001)</td>
<td>0.324 (0.000)</td>
<td>0.282 (0.004)</td>
<td>0.298 (0.000)</td>
</tr>
<tr>
<td>A22</td>
<td>0.262 (0.000)</td>
<td>-0.204 (0.000)</td>
<td>0.014 (0.524)</td>
<td>0.015 (0.869)</td>
<td>-0.016 (0.674)</td>
</tr>
<tr>
<td>B11</td>
<td>-0.603 (0.000)</td>
<td>-0.445 (0.010)</td>
<td>1.059 (0.000)</td>
<td>1.077 (0.000)</td>
<td>-0.179 (0.197)</td>
</tr>
<tr>
<td>B12</td>
<td>-1.237 (0.000)</td>
<td>-1.034 (0.000)</td>
<td>0.273 (0.000)</td>
<td>0.341 (0.000)</td>
<td>-0.855 (0.000)</td>
</tr>
<tr>
<td>B21</td>
<td>-0.374 (0.012)</td>
<td>-0.553 (0.001)</td>
<td>-1.223 (0.000)</td>
<td>-1.428 (0.000)</td>
<td>-0.915 (0.000)</td>
</tr>
<tr>
<td>B22</td>
<td>0.383 (0.016)</td>
<td>0.211 (0.300)</td>
<td>-1.145 (0.000)</td>
<td>-1.095 (0.000)</td>
<td>0.056 (0.651)</td>
</tr>
<tr>
<td>D11</td>
<td>0.198 (0.000)</td>
<td>0.218 (0.000)</td>
<td>0.280 (0.000)</td>
<td>0.332 (0.000)</td>
<td>0.321 (0.000)</td>
</tr>
<tr>
<td>D12</td>
<td>0.413 (0.000)</td>
<td>0.329 (0.000)</td>
<td>0.059 (0.002)</td>
<td>0.142 (0.000)</td>
<td>0.258 (0.000)</td>
</tr>
<tr>
<td>D21</td>
<td>-0.169 (0.000)</td>
<td>-0.103 (0.088)</td>
<td>-0.024 (0.768)</td>
<td>0.166 (0.119)</td>
<td>-0.059 (0.191)</td>
</tr>
<tr>
<td>D22</td>
<td>0.220 (0.000)</td>
<td>0.235 (0.000)</td>
<td>0.214 (0.000)</td>
<td>0.327 (0.000)</td>
<td>0.261 (0.000)</td>
</tr>
<tr>
<td>Log L</td>
<td>-1207.29</td>
<td>-1178.71</td>
<td>-915.22</td>
<td>-870.51</td>
<td>-1175.83</td>
</tr>
<tr>
<td>LR</td>
<td>57.16</td>
<td>89.42</td>
<td>50.04</td>
<td>81.16</td>
<td>44.71</td>
</tr>
</tbody>
</table>

**Notes:**
- LogL is the Log-likelihood function evaluated at the maximum.
- LR is log likelihood ratio statistics, testing for structural changes in variance. The p-values appear in parenthesis.
- The critical value of one $\chi^2$ with four degrees of freedom is .49.