The effect of futures trading activity on the distribution of spot market returns

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Abstract

There is extensive empirical research on the potential volatility transmission from futures to spot market. Rather than just focusing on the effect of futures trading on spot volatility, this paper deals with the contemporaneous relationship between futures trading volume and the overall probability distribution of spot market returns. To disentangle the potential destabilizing effect of futures trading activity from cross-interactions due to price discovery process, futures volume is broken down into two drivers: expected and unexpected trading activity. Then, a non-parametric approach is used to estimate the density function of spot return conditional to both spot and futures trading volume. Empirical evidence using intraday data from the Spanish stock index futures market over the period 2000-2002 is provided. Our empirical findings can be summarized as follows: i) spot market volatility is positively related to spot trading volume, ii) for any given spot trading volume, a significant and positive relationship between unexpected futures trading activity and spot volatility is detected; however no significant relationship arise when expected futures trading volume is considered. These findings are consistent with theoretical models predicting that futures trading activity is not a force behind irrational spot price fluctuations.

1 Introduction

Since their introduction, stock index futures markets have experienced a substantial increase in trading activity. Financial futures contracts are key instruments in portfolio management, as they allow for risk transference. Moreover, derivative markets play an important role within the price discovery process of underlying assets. Stock index futures have relatively lower transaction costs and capital requirements, so the arrival of external information is quickly incorporated into prices as investors' expectations are updated.
The existence of mispricing relative to the cost-of-carry valuation of a stock index futures contract has been well documented in the literature (see Mackinlay and Ramaswamy (1988), Miller et al., (1994), Yadav and Pope (1990, 1994), Bühlert and Kempf (1995) and Fund and Draper (1999), among others). However, either mispricing occurs within the non-profitable arbitrage bounds (see, for example, Lim (1992)) or the adjustment in response to pricing error takes place rapidly (see Taylor et al. (2000), Dwyer et al. (1996), Tse (2001) and Chu and Hsieh (2002)).

Given that spot and futures prices are linked by arbitrage operations, one popular perception is that arbitrage trading activities involving index futures and underlying equities increase stock volatility. Recent episodes of market crash and volatility contagion among countries did not contribute to mitigate such perception. The destabilizing hypothesis has also been supported by academic research. Criticisms of derivative markets argue that lower transaction costs in futures markets attract uninformed speculative order flow, introducing noisy information in the price discovery process, reducing the informativeness of prices and leading to spot price instability (see Cox (1976) and Stein (1987), among others, for relevant theoretical contributions supporting this argument).

Under risk aversion, higher volatility should lead to higher risk premium. This way, transmission of volatility from futures to spot market could raise the required rate of return of investors in the market, leading to a misallocation of resources and a potential loss of welfare in the economy. Hence, empirical work on the relationship between futures trading and spot volatility is of considerable interest for practitioners and specially for regulators.

Even though the relationship between spot volatility and futures trading activity has been extensively analyzed in the literature, empirical evidence is far from conclusive. Following an event study approach, some researchers focus on the behavior of stock index volatility before and after the introduction of the derivative market (see, for example, Antoniou and Holmes (1995), Pericli and Koutmos (1997), Antoniou et al. (1998) and Rahman (2001)). However, as pointed out by Bessembinder and Seguin (1992), the potential volatility change revealed in these studies "need not be solely attributable to the introduction of futures" (p. 2026), but also to other changes in the financial environment during the period examined.

To overcome this problem, Bessembinder and Seguin (1992) proposed an alternative approach which just focuses on a time period subsequent to the introduction of futures market. In particular, to disentangle the potential destabilizing effects of futures market from the cross-interactions involved in the price discovery process, Bessembinder and Seguin (1992) suggest to break down futures trading volume into expected and unexpected components using an ARIMA filter. Once the decomposition is carried out, the contemporaneous relationship between spot volatility and expected (or inforrmationless) futures trading activity is examined (see Illueca and Lafuente (2003), Board et al. (2001) and

\[\text{Indeed, the alleged increase in volatility has led to proposals of closer regulation in the US futures markets during the eighties.}\]
Given the overwhelming empirical evidence supporting that market-wide new information disseminates faster in the futures than in the spot market, and consistent with the hypothesis that information shocks generate trading volume in futures market, a positive relationship between futures volume shocks and spot volatility is expected as traders update their relevant information set. However, the transmission of volatility due to unexpected volume (or volume shocks) should not be considered as a source of instability. Indeed, as stated by William J. Rainer (Chairman of the Commodity Futures Trading Commission, October 28, 1999, 22nd Annual Chicago-Kent College of Law Derivatives), "there is no case for regulating financial futures on the basis of price discovery".

A common feature in the extant literature is the use of a parametric framework to i) estimate spot market volatility and ii) test the effect of futures trading on stock index volatility considering a particular econometric specification relating both variables. However, the stochastic properties of the parameters involved depends on the distributional assumptions of errors. Moreover, as pointed out in Bollerslev et al. (1992), the widely used GARCH models which add futures volume to spot variance equation as an exogenous explanatory variable may suffer from mispecification, leading to biased estimation.

This paper contributes to the literature by using a non-parametric framework to strengthen the Bessembinder and Seguin’s approach. In spite of just restricting the analysis to spot volatility, we generalize the concept of destabilization by considering the effect of futures trading activity on the overall spot return distribution. The Value at Risk of the underlying asset could substantially change even when volatility remains stable relative to futures trading activity. In this paper, kernel smoothing is used to estimate the distribution of spot returns conditional to both spot and futures trading volume. Once the effect of futures trading on the overall spot distribution is analyzed, the impact on any particular moment of the conditional distribution can be tested. In accordance with to the extant literature, we focus on the second order central moment which can be considered as an implied measure of conditional spot volatility that does not assume any particular "news impact surface" (see Kroner and Ng (1998)).

Empirical evidence using 15-minute data from the Spanish stock index futures market is provided from December 1999 to December 2002. Our empirical results show that higher unexpected trading activity in futures markets is associated with higher volatility in spot market, regardless its level of trading activity. However estimated effects of expected futures trading activity on spot volatility are not statistically significant. In sum, our empirical findings do not support the destabilizing hypothesis, at least for the Spanish case.

The rest of the paper is organized as follows. Section II describes the data set and the variables used in the analysis. In section III we present the methodology used to estimate the conditional density function of spot returns. Section IV provides empirical evidence on the relationship between spot returns distribution and both spot and futures trading volume. Finally, section V summarizes and makes concluding remarks.
2 Data

2.1 Description of the variables and sample period

Data on the Ibex 35 spot and futures markets were provided by MEFF RV (Mercado Español de Futuros, Renta Variable) for the period January 17, 2000-December 20, 2002. The number of trading days is equal to 727\(^2\). The intraday trading period considered covers from 9:00 to 17:30\(^3\). Matched data using futures prices and the Ibex 35 index was generated. From matched data we selected 15-minutes prices and then, we generated the percent return series for each market by taking the first difference of the natural logarithm. We excluded overnight returns because they are measured over a longer time period. This procedure finally gave 34 return observations for each trading day. We associated the volume traded \(^4\) within the corresponding 15-minutes trading interval to each spot and futures market return. Overall, we obtained 24,718 return and trading volume observations for each market.

Since the nearest to maturity contract is systematically the most actively traded, only data for the nearby futures contract was used. The point in time at which the current contract is rolled to the next is selected according to futures market depth. Ma et al. (1992) show that the conclusions drawn from three common empirical tests of futures markets (namely, a) risk-return combinations of a buy-and-hold trading strategy, b) serial dependence between returns and c) daily effects in the pay-off distributions) are not robust to the choice of method for rolling over futures contracts. The methods considered involve combinations of alternative dates at which the current contract is rolled as well as price adjustments. Figure 1 (Appendix 2) shows the intraday average trading volume within the expiration date, revealing that at 16:30 the next maturity becomes the higher volume contract. From that moment on, returns are computed using prices that correspond to such maturity.

2.2 Decomposition of detrended volume

To detrend spot and futures volume series, we first partitioned the intraday trading period into eight intervals according to the following time sequence: [9:00 10:00 11:00 12:00 13:00 14:00 15:00 16:00 17:30]\(^5\). For each market and each interval we formed stationary time series of trading volume by using a centered moving average (see Fung and Patterson (1999) and Campbell et al. (1993)).

\(^2\) Data from: a) 07/11/2000 and b) 11/02/2001 were not available in the Meff Renta Variable data set, and were therefore not included.

\(^3\) Before January 17, 2000, Ibex 35 futures contracts were traded from 9:30 to 17:00.

\(^4\) Trading volume is measured in millions of euros.

\(^5\) We performed such time partition to take into account the intraday U-shape curve in trading volume.
\[ V_{t-1,t} = \frac{TV_{t-1,t}}{\frac{1}{N} \sum_{j=-N}^{N-1} TV_{t-1+j,t+j}}, \]  

(1)

where \( TV_{t-1,t} \) is the trading volume between \( t-1 \) and \( t \), \( N \) is the number of observations used to capture the trend of the series. We consider \( N = 21 \) for the seven hourly intervals generated from 9:00 to 16:00. For the last interval (16:00 to 17:30) we set \( N = 31 \). This volume measure produces a detrended time series that incorporates the change in the short-run movement in trading volume. Table 1 provides the Augmented Dickey fuller test on the detrended volume series for both spot and futures market, thus corroborating that they are stationary.

For each trading interval we decompose volume into predictable and unpredictable components by using a bivariate Vector Autoregression:

\[
\begin{pmatrix}
V_{\text{spot}}
t \\
V_{\text{fut}}
t
\end{pmatrix} = C + \sum_{j=1}^{p} \Psi_j \begin{pmatrix}
V_{\text{spot}}
t-p \\
V_{\text{fut}}
t-p
\end{pmatrix} + U_t
\]

(2)

where \( U_t \sim N (0, \Sigma) \), \( \Psi_j \) are \( 2 \times 2 \) matrices that capture the impact of past trading volume in both markets. The fitted values from 2 are interpreted as the informationless trading, while the residuals of the model are interpreted as the innovation in trading activity in each market. The lag structure used involves past information corresponding to the three previous days. Table 2 reports the test for joint significance of each group of lags. Significant cross interactions between trading volume are detected, suggesting that a univariate ARIMA model would not be adequate to filter raw series in order to identify expected and unexpected trading volume variables.

### 3 Methodology

To test the effect of trading activity on spot volatility, two approaches have been widely proposed in the literature: a) conventional regression analysis, and b) GARCH models. The first approach is a two-stage procedure. Initially, an estimation of volatility is performed by means of squared returns, Garman-Klass statistic (Garman-Klass (1980)), among many others. An econometric specification involving trading activity and volatility variables is then estimated. The second approach is a one stage procedure which allows for the incorporation of the effect of trading volume in the estimation of market volatility.

Assuming without loss of generality that spot return \( (R_s) \) has zero mean, parametric approaches seek to test whether in

\[
E \left( R_{s,t}^2 | TA_{f,t}, R_{s,t-j}^2, j > 0 \right) = \Phi \left( R_{s,t-j}^2, j > 0 \right) + \gamma TA_{f,t}
\]

(3)

\[ \Phi \] | Empirical results reported in the paper are qualitatively robusts to alternative specifications of the VAR model \((p = 24, p = 36)\).
the coefficient $\gamma$ is not significant at conventional levels, where $\Phi$ is a parametric function and $TA_{f,t}$ is a variable that refers to futures trading activity (trading volume, open interest and related).

However, there is no reason why researchers should be only interested in the conditional variance of spot returns. More generally, and specially under departures from normality, the researcher might focus on the behavior of the overall spot return distribution. In this paper, we use a kernel estimation procedure to analyze the effect of futures trading activity on spot prices. Kernel estimation is a non-parametric technique for estimating the joint density of a set of random variables (Silverman, 1986). A kernel estimator of a bivariate density is

$$f_M(X; H) = \frac{1}{T} \sum_{i=1}^{T} K_H(X - X_i)$$  

(4)

where $T$ is the sample size, $X_i$ denotes the $i$-th sample observation of a two-dimensional variable, $K_H$ is a function involving the Kernel function ($K$) and the smoothing matrix ($H$), with the following general form:

$$K_H(Z) = |H|^{-\frac{\gamma}{2}} K \left( H^{-\frac{1}{2}} Z \right)$$  

(5)

The Epanechnikov kernel function (Epanechnikov (1969)) is used:

$$K(x) = \{ \begin{array}{ll} \frac{3}{4} (1 - x'x) & \text{if } x'x < 1 \\ 0 & \text{if } x'x \geq 1 \end{array}$$

Relative to the smoothing parameters, the window width matrix is computed according to the plug-in-solve-the equation method suggested by Wand and Jones (1994).

To implement the objective of the paper, we first estimate the joint probability distribution of the bivariate ($R_s, Volume$) vector. Secondly, the implied unconditional marginal density function of volume is obtained from the bivariate density. The density function of spot returns conditional to trading volume is then computed as the ratio between the joint density function and the implied marginal density of trading volume.

The final outcome is similar to a multidimensional histogram. Just as with an histogram, for each point in the sample a "block" of volume $\frac{1}{T}$ is added. However, two key differences must be highlighted: i) when the Epanechnikov kernel is considered, the "blocks" are not rectangular, and ii) they are centered at each data point rather than at the center of a fixed number of bins.

\footnote{For operational purposes, all the variables involved in the kernel estimation are re-scaled dividing by their standard deviation.}

\footnote{The use of the Epanechnikov kernel leads to minimize the MISE. However, other reasonable kernel functions could have been used (gaussian, rectangular or triangular, among others). Previous literature on kernel estimation suggests that these functions give almost optimal results.}
4 Empirical Results

4.1 Spot return and spot trading volume

Following Bessembinder and Seguin (1992), we initially investigate the contemporaneous relationship between spot return distribution and spot trading volume. Figure 2 depicts the density function of spot returns conditional to total spot trading volume. As it is readily apparent, the conditional density functions of spot returns vary in accordance with spot volume. In particular, the probabilistic mass spreads as trading activity augments, suggesting a positive relationship between price fluctuation and market depth in spot market. The breakdown of total spot volume into expected and unexpected components provides additional insights about the nature of the linkages between spot returns and spot trading activity. Figures 3 and 4 show the density functions of spot returns conditional to informationless volume and volume shocks, respectively. Interestingly enough, while Figure 4 replicates the pattern of the conditional density shown in Figure 2, the conditioning on expected trading volume seems to be less relevant. Table 3 reports the results of testing the null hypothesis of stochastic independence between spot return distribution and spot trading volume (total, expected and unexpected) distribution. Empirical values systematically lead to reject the null at 1% significance level.

These findings are consistent with previous research showing a positive correlation between volume and absolute returns in equity markets (see Karpoff, 1987). One possible explanation is the information flow hypothesis. Since price changes per unit of calendar time are the sum of the prices changes occurring during such period, if it is assumed that a) prices evolve when new information arrives at the market and b) the number of information arrivals is random, a positive correlation is expected between volume and absolute returns as volume is positively correlated with the number of information arrivals to the market.

In sum, the previous findings reveal that spot trading volume is a relevant variable to explain the behavior of spot price changes, suggesting that volume of trade is a good proxy to represent the rate of information flow in the market. Under the assumption that the number of information arrivals is an autocorrelated random variable, volume should contribute significantly in explaining the GARCH effects in stock returns. Indeed, Lamoureux and Lastrapes (1990) provide empirical evidence showing that the parameter estimates of the GARCH model become insignificant when volume of trade is used in the conditional variance of stock returns.

4.2 Spot return and total futures trading activity

In this section we proceed to analyze the relationship between futures trading volume and spot return distribution. Based on the foregoing empirical findings, the effect of spot trading activity on the distribution of spot returns should be taken into account. In particular, the null hypothesis that we are going to test is:
\[ H_0 : g(R_s|TV_s) = g(R_s|TV_s, TV_f) \]

where \( g \) refers to the density function, \( R \) and \( TV \) denote returns and total volume respectively, and subindexes \( s \) and \( f \) refer to spot and futures markets.

To implement the analysis we partition the total sample of the \((R_s, TV_s, TV_f)\) tridimensional variable into five equally sized groups according to .20-th quantiles of \( TV_f \). Let us denote each of the five subsamples of the bivariate \((R_s, TV_s)\) variable as \((R_s, TV_s)^j\) where \( j \) denotes that the subsample corresponds to the \([(j - 1) * 20, j * 20]\) -th quantile of \( TV_f \).

Figures 5 to 7 depict the density function of spot return conditional to spot trading volume for \( j = 1, 3, 5 \)\(^9\). Two interesting aspects arise from these figures: a) only for the fifth subsample (high futures trading activity) the conditional density of spot returns is similar to that reported in Figure 2 corresponding to \( g(R_s|TV_s) \), and b) the conditional density of spot returns does not remain unchanged as futures trading activity augments. Given a particular spot market depth, higher futures trading activity is associated with larger tails of spot returns distribution. To formally test the foregoing null hypothesis of equality between conditional distributions, a goodness-of-fit test is performed. Empirical values of the chi-squared test are reported in table 5. In all cases, the null hypothesis of equality is rejected at 1% significance level. In sum, our empirical findings reveal that both spot and futures trading volumes are relevant variables to explain the distribution of spot price changes.

### 4.2.1 The effect of futures trading activity: price discovery or destabilization?

As mentioned above, any transaction in the derivative market should not be considered as a potential source of instability. Unexpected trading volume is related to the information arrivals to the market, while the expected component can be considered as the natural activity in the derivative market, that is, futures trading volume when no relevant new information arrives at the market.

There is conclusive evidence in the literature on the leadership of futures markets over the price discovery process. The arrival of new information tends to disseminate faster in the futures markets, inducing spot price changes through arbitrage operations. This way, as long as shocks affect both markets in the same direction, a positive correlation between unexpected volume and absolute spot returns is expected.

As pointed out by Bessembinder and Seguin (1992), the destabilizing hypothesis concerns the relationship between expected futures trading and spot market returns. Now, the relevant null hypothesis is:

\[ H_0 : g(R_s|TV_s) = g(R_s|TV_s, EV_f) \]

\(^9\)To save space the conditional density functions corresponding to \( j = 2, 4 \) are not reported in the paper. They are available from the authors upon request. Anyway, the reported figures allow to properly observe the behavior of the conditional density function under alternative (low, medium and high) futures trading activity.
where $EV_f$ denotes the expected futures volume.

Figures 8 to 10 report the conditional density of spot returns for alternative (low, medium, high) levels of expected futures trading activity. The visual inspection of these figures does not reveal any substantial difference between them. Indeed, conditional densities are quite similar to that depicted in Figure 2 corresponding to $g(R_s|TV_s)$. Table 6 presents the empirical values of the goodness-of-fit test for the previous null hypothesis. As expected, the null hypothesis of equality between both conditional density functions is not rejected at conventional levels, corroborating that informationless futures volume does not incorporate relevant information to explain spot market returns. This finding does not support the existence of destabilizing effects from the Ibex 35 futures market to spot index.

Relative to the impact of futures volume shocks, the relevant null hypothesis is:

$$H_0 : g(R_s|TV_s) = g(R_s|TV_s, UV_f)$$

where $UV_f$ denotes the unexpected futures volume.

Figures 11 to 13 depict the density function of spot returns conditional to different levels of unexpected futures trading volume. In contrast to the pattern observed in Figures 8 to 10, the conditional densities vary now with the level of unanticipated futures trading. Specifically, the higher unexpected futures trading, the higher dispersion of conditional spot returns, suggesting that a volatility transmission to the spot market takes place when futures prices react to the arrival of new information. However, as already mentioned, this pattern is to be expected in the price discovery process. These differences in the conditional density functions are corroborated by the empirical values of the chi-squared statistic to test the null hypothesis of equality between conditional distributions (see Table 7), which systematically lead to reject the null at 1% significance level.

In sum, our empirical findings for the Spanish market reveal that futures trading activity is a significant variable to explain the density function of spot returns conditional to spot trading volume. However, this relationship is solely attributable to the price discovery function of the futures market, that is, no destabilizing effects are detected.

Once the impact on the overall distribution is analyzed, a partial study concerning the moments of the probability distribution could provide additional insights. Indeed, as the conditional distribution changes, their moments should change as well. In accordance with previous research in the literature, the particular case of the second order central moment is carried out in the following section.

### 4.2.2 Conditional volatility analysis

From the density function of spot return conditional to spot trading volume, the conditional second order central moment for alternative levels of futures trading volume is:
\[
\text{Var} (R_s|TV_s, V_f) = \int_{-\infty}^{+\infty} (R_s - E(R_s|TV_s, V_f))^2 g(R_s|TV_s, V_f) dR_s
\]

where \( V_f \) is a variable that refers to the nature of futures trading volume (total, expected and unexpected).

Figure 14 shows the variance of spot return conditional to spot volume for each of the five subsamples drawn according to the .20-th quantiles of total futures trading volume. As expected from the shape of previously reported conditional density functions, higher futures activity is associated with higher volatility for any spot market depth. A Kolmogorov-Smirnov test is performed to statistically corroborate such pattern. In particular, the null and the alternative hypothesis are:

\[
\begin{align*}
    H_0 &: F(\sigma_s^2|TV_s, TV_j, f) = F(\sigma_s^2|TV_s, TV_{j+1}, f) \\
    H_1 &: F(\sigma_s^2|TV_s, TV_j, f) > F(\sigma_s^2|TV_s, TV_{j+1}, f)
\end{align*}
\]

where \( \sigma_s^2 \) denotes the conditional spot variance and \( F \) refers to the cumulative distribution function and \( TV_{j,f} \) is the total futures volume that corresponds to the \( [(j-1)*20, j*20] \)-th quantile. Table 8 reports the empirical values of the test, which confirm a positive relationship between conditional spot volatility and total futures trading activity.

Figure 15 and 16 depict the conditional spot volatility under different levels of expected and unexpected futures trading volume. While spot volatility remains unchanged as informationless futures volume rises, a positive relationship between spot price fluctuations and futures trading activity arises when the unexpected component is considered. Tables 9 and 10 provide the empirical values of the corresponding Kolmogorov-Smirnov test for the expected and unexpected futures volume, respectively. In both cases, the results of the test are consistent with the pattern observed in the conditional spot volatility.

### 5 Conclusions

This paper provides empirical evidence on the destabilizing hypothesis in the Spanish stock index futures market. In spite of just focusing on the effect of futures trading on spot volatility, we propose a more general approach which consists of examining the contemporaneous relationship between futures trading activity and the overall probability distribution of spot market returns.

Using 15-minute intraday data covering the period 2000-2002, a non-parametric kernel smoothing procedure is applied to estimate the conditional density function of spot returns conditional to spot volume. Consistent with the information flow hypothesis, spot volume significantly contributes to explain spot price fluctuations.

To test the effect of futures trading on the distribution of spot returns, we reestimate the conditional density function of spot returns under different levels of expected and unexpected futures trading volume.

\[10 \text{The implicit conditional variance is computed for each of the .02-th quantile of the spot volume.}\]
of futures trading volume (low, medium and high activity). Empirical results reveal that the conditional density function of spot returns depends on futures trading. In particular, higher futures trading leads to fatter-tailed conditional distributions of spot returns.

To investigate whether or not such tail behavior is solely related to the price discovery function of futures market, we break down the total futures volume into unexpected and expected components using VAR methodology. The effect of unexpected futures volume is similar to that of total trading volume. But, interestingly enough, the estimated conditional density function of spot returns remains unchanged under different levels of expected futures trading volume.

In accordance with previous research in the literature, a particular analysis of the conditional second order central moment (conditional spot volatility) is also performed. Consistent with previous findings, the expected futures trading volume does not contribute to explain the conditional spot volatility of spot returns. However, the arrival of new information to the futures market is positively correlated with conditional spot volatility.

In summary, contrary to the traditional view of futures trading, this research provides no empirical support for futures market being a force behind spot destabilization. Therefore, there is no justification for regulatory initiatives to limit futures trading based on the assumption that futures trading tends to destabilize spot market prices, at least in the Spanish stock index futures market.

References


Appendix 1 (Tables)

Table 1. Unit root test for stock and futures market volume series

<table>
<thead>
<tr>
<th>Trading interval</th>
<th>Spot</th>
<th>Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00-10:00</td>
<td>-23.82</td>
<td>-22.48</td>
</tr>
<tr>
<td>10:00-11:00</td>
<td>-23.11</td>
<td>-23.75</td>
</tr>
<tr>
<td>11:00-12:00</td>
<td>-23.59</td>
<td>-23.39</td>
</tr>
<tr>
<td>13:00-14:00</td>
<td>-23.84</td>
<td>-22.56</td>
</tr>
<tr>
<td>14:00-15:00</td>
<td>-24.73</td>
<td>-23.02</td>
</tr>
<tr>
<td>15:00-16:00</td>
<td>-24.43</td>
<td>-22.96</td>
</tr>
<tr>
<td>16:00-17:30</td>
<td>-25.63</td>
<td>-24.81</td>
</tr>
</tbody>
</table>

The table reports the results of the test of the null hypothesis $H_0 : \rho = 0$ from the regression of the form:

$$\Delta V_t = \rho V_{t-1} + \alpha + \sum_{j=1}^{p} \Delta V_{t-j} + \varepsilon_t$$

where the number of lags ($p = 12$) is chosen in order to ensure no significant residual autocorrelation. The MacKinnon critical values for rejection of hypothesis of a unit root at the 1% and 5% significance level are -3.4421 and -2.8660, respectively.

Table 2. Test of joint significance in the VAR model

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$V_{spot}$</th>
<th>$V_{fut}$</th>
<th>$V_{spot}$</th>
<th>$V_{fut}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group of regressors</td>
<td>$V_{spot}$</td>
<td>$V_{fut}$</td>
<td>$V_{spot}$</td>
<td>$V_{fut}$</td>
</tr>
<tr>
<td>9:00 - 10:00</td>
<td>223.8</td>
<td>145.2</td>
<td>178.5</td>
<td>276.5</td>
</tr>
<tr>
<td>10:00 - 11:00</td>
<td>262.8</td>
<td>66.2</td>
<td>283.8</td>
<td>148.1</td>
</tr>
<tr>
<td>11:00 - 12:00</td>
<td>302.0</td>
<td>54.0</td>
<td>298.7</td>
<td>125.5</td>
</tr>
<tr>
<td>12:00 - 13:00</td>
<td>362.8</td>
<td>87.9</td>
<td>290.0</td>
<td>181.9</td>
</tr>
<tr>
<td>13:00 - 14:00</td>
<td>263.0</td>
<td>60.0</td>
<td>320.5</td>
<td>135.8</td>
</tr>
<tr>
<td>14:00 - 15:00</td>
<td>381.7</td>
<td>35.1</td>
<td>347.0</td>
<td>182.3</td>
</tr>
<tr>
<td>15:00 - 16:00</td>
<td>343.5</td>
<td>115.7</td>
<td>378.4</td>
<td>254.9</td>
</tr>
<tr>
<td>16:00 - 17:30</td>
<td>693.7</td>
<td>121.9</td>
<td>353.4</td>
<td>300.4</td>
</tr>
</tbody>
</table>

Note: Empirical values of the Wald test systematically lead to reject at conventional levels the null hypothesis that all the coefficients corresponding to each group of regressors are equal to zero.
Table 3. Testing stochastic independence between spot return and spot volume distributions

<table>
<thead>
<tr>
<th>Null Hypothesis: Independence between</th>
<th>$\chi^2_{(r-1)^2}$</th>
<th>p-value</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spot return and Total spot Volume</td>
<td>2611.2 (0.00)</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Spot return and Expected spot Volume</td>
<td>1706.6 (0.00)</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Spot return and Unexpected spot Volume</td>
<td>2512.5 (0.00)</td>
<td>28</td>
<td></td>
</tr>
</tbody>
</table>

Note: To implement this test a discrete version of the conditional density function is required. A partition of both supports (spot return and volume) into $r$ equally sized groups is considered. The chi-squared statistic to test the null hypothesis of stochastic independence is:

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \left( \frac{N_{ij} - N_i N_j}{N_i N_j} \right)^2$$

where $N_{ij}$ is the number of observations within the $i$-th group of returns and the $j$-th group of volume. The use of the asymptotic distribution is suitable when $N_{ij} \geq 5$. To maximize the power of the test, we consider the maximum number of groups ($r$) subject to the previous constraint.

Table 4. Correlation coefficients between spot volatility and spot volume

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Total Volume</th>
<th>Expected Volume</th>
<th>Unexpected Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td>0.96(*)</td>
<td>0.72(*)</td>
<td>0.88(*)</td>
</tr>
<tr>
<td>Spearman</td>
<td>1.00(*)</td>
<td>0.63(*)</td>
<td>1.00(*)</td>
</tr>
</tbody>
</table>

Note: (*) denotes statistical at 1% level.

Table 5. Testing the equality between conditional distributions of spot returns for alternative levels of total futures trading volume

<table>
<thead>
<tr>
<th>Null Hypothesis:</th>
<th>$\chi^2_{(r-1)^2}$</th>
<th>p-value</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g(R_s</td>
<td>T_{V_j}) = g(R_s</td>
<td>T_{V_j}^1)$</td>
<td>933.5 (0.00)</td>
</tr>
<tr>
<td>$g(R_s</td>
<td>T_{V_j}) = g(R_s</td>
<td>T_{V_j}^2)$</td>
<td>442.5 (0.00)</td>
</tr>
<tr>
<td>$g(R_s</td>
<td>T_{V_j}) = g(R_s</td>
<td>T_{V_j}^3)$</td>
<td>429.3 (0.00)</td>
</tr>
<tr>
<td>$g(R_s</td>
<td>T_{V_j}) = g(R_s</td>
<td>T_{V_j}^4)$</td>
<td>383.3 (0.00)</td>
</tr>
<tr>
<td>$g(R_s</td>
<td>T_{V_j}) = g(R_s</td>
<td>T_{V_j}^5)$</td>
<td>369.8 (0.00)</td>
</tr>
</tbody>
</table>

Note: $T_{V_j}$ refers to the subsample of $(R_s,T_{V_j})$ that corresponds to the $[(j-1) \cdot 20, j \cdot 20]$-th quantile of total futures volume. To implement this test a discrete version of the conditional density function is required. A partition of both supports (spot return and volume) into $r$ equally sized groups is considered. The chi-squared statistic to test the goodness-of-fit is:

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \left( \frac{f_{ik} - p_{ik}}{p_{ik}} \right)^2$$

where $p_{ik} = \frac{N_{ik}}{T}$, $T$ is the sample size and $f_{ik}$ is the number of observations within the $i$-th group of returns and the $k$-th group of spot volume within the subsample corresponding to $[(j-1) \cdot 20, j \cdot 20]$-th quantile of total futures volume. The use of the asymptotic distribution is suitable when $f_{ik} \geq 5$. To maximize the power of the test, we consider the maximum number of groups ($r$) subject to the previous constraint.
maximum number of groups is suitable when \( \text{quantile of group of spot volume within the subsample corresponding to} \) is the number of observations within the \( \text{considered. The chi-squared statistic to test the goodness-of-fit is:} \)

\[
\chi^2_{(r-1)^2} = \sum_{i=1}^{r} \sum_{j=1}^{r} \left( \frac{f_{ij} - p_{ik}}{p_{ik}} \right)^2 \]

where \( p_{ik} = \frac{N_{ik}}{N_{i}} \) or \( T \) is the sample size and \( f_{ij} \) is the number of observations within the \( i-th \) group of returns and the \( k-th \) group of spot volume within the subsample corresponding to \( \text{of expected futures volume. To implement this test a discrete version of the conditional density function is required. A partition of both supports (spot return and volume) into} \( r \) equally sized groups is considered. The \( \chi^2 \) statistic to test the goodness-of-fit is:

\[
\chi^2_{(r-1)^2} = \sum_{i=1}^{r} \sum_{k=1}^{r} \left( \frac{f_{ik} - p_{ik}}{p_{ik}} \right)^2 \]

where \( p_{ik} = \frac{N_{ik}}{N_{i}} \) or \( T \) is the sample size and \( f_{ik} \) is the number of observations within the \( i-th \) group of returns and the \( k-th \) group of spot volume within the subsample corresponding to \( \text{of expected futures volume. The use of the asymptotic distribution is suitable when} \( f_{ik} \geq 5 \). To maximize the power of the test, we consider the maximum number of groups \( (r) \) subject to the previous constraint.

### Table 6. Testing the equality between conditional distributions of spot returns for alternative levels of expected futures trading volume

| Null Hypothesis: \( g(R_s|TV_s) = g(R_s|TV_e) \) | \( \chi^2_{(r-1)^2} \) | p-value | \( r \) |
|-----------------------------------------------|-----------------|---------|------|
| \( g(R_s|TV_s) = g(R_s|TV_e) \) | 108.3 | (0.79) | 12 |
| \( g(R_s|TV_s) = g(R_s|TV_e) \) | 96.9 | (0.95) | 12 |
| \( g(R_s|TV_s) = g(R_s|TV_e) \) | 142.0 | (0.53) | 13 |
| \( g(R_s|TV_s) = g(R_s|TV_e) \) | 168.8 | (0.92) | 15 |
| \( g(R_s|TV_s) = g(R_s|TV_e) \) | 173.1 | (0.88) | 15 |

Note: \( TV_e \) refers to the subsample of \( (R_s, TV_s) \) that corresponds to the \( [(j-1) \ast 20, j \ast 20] \)th quantile of expected futures volume. To implement this test a discrete version of the conditional density function is required. A partition of both supports (spot return and volume) into \( r \) equally sized groups is considered. The \( \chi^2 \) statistic to test the goodness-of-fit is:

\[
\chi^2_{(r-1)^2} = \sum_{i=1}^{r} \sum_{k=1}^{r} \left( \frac{f_{ik} - p_{ik}}{p_{ik}} \right)^2 \]

where \( p_{ik} = \frac{N_{ik}}{N_{i}} \) or \( T \) is the sample size and \( f_{ik} \) is the number of observations within the \( i-th \) group of returns and the \( k-th \) group of spot volume within the subsample corresponding to \( \text{of expected futures volume. The use of the asymptotic distribution is suitable when} \( f_{ik} \geq 5 \). To maximize the power of the test, we consider the maximum number of groups \( (r) \) subject to the previous constraint.

### Table 7. Testing the equality between conditional distributions of spot returns for alternative levels of unexpected futures trading volume

| Null Hypothesis: \( g(R_s|TV_s) = g(R_s|TV_u) \) | \( \chi^2_{(r-1)^2} \) | p-value | \( r \) |
|-----------------------------------------------|-----------------|---------|------|
| \( g(R_s|TV_s) = g(R_s|TV_u) \) | 773.0 | (0.00) | 17 |
| \( g(R_s|TV_s) = g(R_s|TV_u) \) | 331.1 | (0.00) | 16 |
| \( g(R_s|TV_s) = g(R_s|TV_u) \) | 330.5 | (0.00) | 17 |
| \( g(R_s|TV_s) = g(R_s|TV_u) \) | 273.2 | (0.00) | 13 |
| \( g(R_s|TV_s) = g(R_s|TV_u) \) | 360.9 | (0.00) | 9 |

Note: \( TV_u \) refers to the subsample of \( (R_s, TV_s) \) that corresponds to the \( [(j-1) \ast 20, j \ast 20] \)th quantile of unexpected futures volume. To implement this test a discrete version of the conditional density function is required. A partition of both supports (spot return and volume) into \( r \) equally sized groups is considered. The \( \chi^2 \) statistic to test the goodness-of-fit is:

\[
\chi^2_{(r-1)^2} = \sum_{i=1}^{r} \sum_{k=1}^{r} \left( \frac{f_{ik} - p_{ik}}{p_{ik}} \right)^2 \]

where \( p_{ik} = \frac{N_{ik}}{N_{i}} \) or \( T \) is the sample size and \( f_{ik} \) is the number of observations within the \( i-th \) group of returns and the \( k-th \) group of spot volume within the subsample corresponding to \( \text{of unexpected futures volume. The use of the asymptotic distribution is suitable when} \( f_{ik} \geq 5 \). To maximize the power of the test, we consider the maximum number of groups \( (r) \) subject to the previous constraint.
Table 8. Testing the equality between conditional volatility distributions for alternative levels of total futures trading volume

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>( F(\sigma^2_{TV_n, TV_{1,f}}) = F(\sigma^2_{TV_n, TV_{2,f}}) )</th>
<th>( KS )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 )</td>
<td>( F(\sigma^2_{TV_n, TV_{1,f}}) = F(\sigma^2_{TV_n, TV_{2,f}}) )</td>
<td>.959 (***))</td>
</tr>
<tr>
<td>( H_1 )</td>
<td>( F(\sigma^2_{TV_n, TV_{1,f}}) &gt; F(\sigma^2_{TV_n, TV_{2,f}}) )</td>
<td></td>
</tr>
</tbody>
</table>

Note: (**) and (***)) denotes the rejection of the null hypothesis at 10% and 1% significance level, respectively.

Table 9. Testing the equality between conditional volatility distributions for alternative levels of expected futures trading volume

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>( F(\sigma^2_{TV_n, EV_{1,f}}) = F(\sigma^2_{TV_n, EV_{2,f}}) )</th>
<th>( KS )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 )</td>
<td>( F(\sigma^2_{TV_n, EV_{1,f}}) = F(\sigma^2_{TV_n, EV_{2,f}}) )</td>
<td>.163</td>
</tr>
<tr>
<td>( H_1 )</td>
<td>( F(\sigma^2_{TV_n, EV_{1,f}}) \neq F(\sigma^2_{TV_n, EV_{2,f}}) )</td>
<td></td>
</tr>
</tbody>
</table>

Note: None of the tests lead to reject the null hypothesis at 1% significance level.

Table 10. Testing the equality between conditional volatility distributions for alternative levels of unexpected futures trading volume

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>( F(\sigma^2_{TV_n, UV_{1,f}}) = F(\sigma^2_{TV_n, UV_{2,f}}) )</th>
<th>( KS )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 )</td>
<td>( F(\sigma^2_{TV_n, UV_{1,f}}) = F(\sigma^2_{TV_n, UV_{2,f}}) )</td>
<td>.737 (***))</td>
</tr>
<tr>
<td>( H_1 )</td>
<td>( F(\sigma^2_{TV_n, UV_{1,f}}) &gt; F(\sigma^2_{TV_n, UV_{2,f}}) )</td>
<td></td>
</tr>
</tbody>
</table>

Note: (**) and (***)) denotes the rejection of the null hypothesis at 10% and 1% significance level, respectively.
Appendix 2 (Figures)

Figure 1. Average intraday futures trading volume within time to maturity
Figure 2. Density function of spot return conditional to total spot volume

Figure 3. Density function of spot return conditional to expected spot volume

Figure 4. Density function of spot return conditional to unexpected spot volume
Figure 5. Density function of spot return conditional to total spot volume when the total contemporaneous futures volume ∈ [0-20]-th quantile

Figure 6. Density function of spot return conditional to total spot volume when the total contemporaneous futures volume ∈ [40-60]-th quantile

Figure 7. Density function of spot return conditional to total spot volume when the total contemporaneous futures volume ∈ [80-100]-th quantile
Figure 8. Density function of spot return conditional to total spot volume when the expected contemporaneous futures volume $\in [0-20]$-th quantile

Figure 9. Density function of spot return conditional to total spot volume when the expected contemporaneous futures volume $\in [40-60]$-th quantile

Figure 10. Density function of spot return conditional to total spot volume when the expected contemporaneous futures volume $\in [80-100]$-th quantile
Figure 11. Density function of spot return conditional to total spot volume when the unexpected contemporaneous futures volume $\in \{0-20\}$-th quantile

Figure 12. Density function of spot return conditional to total spot volume when the unexpected contemporaneous futures volume $\in \{40-60\}$-th quantile

Figure 13. Density function of spot return conditional to total spot volume when the unexpected contemporaneous futures volume $\in \{80-100\}$-th quantile
Figure 14. Conditional variance of spot returns under different levels of total futures trading volume (\(TFTV\))

Figure 15. Conditional variance of spot returns under different levels of expected futures trading volume (\(EFTV\))
Figure 16. Conditional variance of spot returns under different levels of unexpected futures trading volume (UFTV)